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RENEGOTIATION DESIGN UNDER SYMMETRIC INFORMATION

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No. 522 June 1989

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PRELIMINARY

COMMENTS WELCOME

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ABSTRACT

It has been emphasized that, when some relevant variables are not contractible, the possibility of contract renegotiation may prevent achieving desirable allocations which could be implemented otherwise. We analyze a situation where renegotiation is always possible, but contracts can influence the renegotiation process ; we show that some flexibility in the choice of this process allows to achieve efficiency in a variety of situations, including optimal risk-sharing and investment decisions.

I. INTRODUCTION

This paper analyzes aspects of contract renegotiation in situations where some information, although observable by the contracting parties, cannot be included in the contracts. This has been considered for example to assess the risk of underinvestment in transaction-specific capital by Williamson (1979) and Klein, Crawford and Alchian (1978). Grossman and Hart (1986) have extended these insights to formally investigate the costs and benefits of vertical integration.

In these papers, contracts are assumed *ex ante* to be very simple. In their analysis of underinvestment in a buyer-seller relationship, Hart and Moore (1988) broaden the focus to consider optimal contracts, with verifiable messages designed to influence the outcome of the relationship. They show that when the parties cannot precommit to *ex post* inefficient outcomes, the unverifiability of specific investment and of the state of nature (valuation and cost parameters) may lead again to underinvestment.

Hart and Moore deal with renegotiation using a specific extensive form, which constrains the parties' ability to achieve optimal outcomes. In their framework, once the parties have invested and learned the state of nature, there is a phase where verifiable (i.e. contractible) messages are sent, followed by unverifiable messages that impose *ex post* efficiency in equilibrium. Under this set of assumptions, underinvestment cannot be avoided.

The issue of unverifiable information has also been analyzed in the mechanism-design literature, with the work on Nash implementation (Maskin (1977, 1985)) and subgame-perfect implementation (Moore and Repullo (1988)). The requirement of *ex post* efficiency through renegotiation has been analyzed by Maskin and Moore (1988). Their approach is to add to any mechanism an exogenous axiomatic renegotiation function which transforms any outcome into an *ex post* efficient one that Pareto-dominates it. They derive necessary and

sufficient Nash- and subgame-perfect implementation conditions for two and more than two agents. Green and Laffont (1988) have recently taken a similar exogenous renegotiation function approach to analyze the underinvestment problem. Like Hart and Moore, they derive an underinvestment result. They also provide an example where optimal risk sharing (without investment) cannot be achieved (in contrast to Hart and Moore who, however, worked with a no-wealth-effect assumption).

In this paper, we give another look at this problem from the point of view of renegotiation design. We try to see whether unverifiability is enough per se to generate inefficient outcomes *ex ante*, when the parties are allowed to design optimal renegotiation rules which, while not allowing them to commit to *ex post* inefficient allocations, however permit them to influence the outcome of the renegotiation process. We present axiomatic conditions on renegotiation design that are sufficient to overcome the problems created by unverifiability, and we also show how these conditions emerge naturally in the standard strategic bargaining framework under complete information. In fact, our general focus is related to Williamson (1985)'s view on the role of contracts in the design of a governance structure to deal with contingencies not explicitly prespecified in the original agreement, in part because of unverifiability.

Our focus on renegotiation design will allow us to derive insights concerning the underinvestment debate opened in this general perspective, and to argue that first-best risk sharing and optimal investment levels can be achieved in a wide variety of situations.

The outline of the paper is as follows. Section II focuses on renegotiation design per se. After a presentation of the economic problem under analysis, we first present the three assumptions on renegotiation we use in this paper: the requirement of *ex post* efficiency, the ability of the parties to specify default options which provide lowers bounds on their

payoffs, and the ability to distribute all the bargaining power to either party in the renegotiation stage (i.e. to maintain the other party on its default option payoff). We then show how this set of axioms emerges naturally when we consider renegotiation as a strategic sequential bargaining process with default options (as in Binmore, Rubinstein and Wolinsky (1986)). In fact, allocating all bargaining power to one party can be achieved through the use of "hostages", which take the form of an initial transfer refundable without interest upon agreement.

Sections III to V then use our assumptions on renegotiation design to generate positive implementation results. Section III considers pure risk sharing, and goes beyond the assumption of absence of wealth effects, for which Hart and Moore have shown the first best to be implementable. It provides sufficient conditions for which renegotiation design as defined in Section II can overcome the negative result presented by Green and Laffont. Sections IV and V then address the case of investment, respectively with and without risk neutrality. Once again, our assumptions allow us to generate *ex ante* efficiency, with moreover a very simple contract in the absence of risk aversion. In Section IV, we present a discussion of the insights provided by these results for the general underinvestment problem under *ex post* opportunism.

Finally, Section VI provides concluding remarks and directions for research.

II. RENEGOTIATION DESIGN

1. Basic framework

We consider a trading relationship between two agents. The relevant trading quantitites are summarized by a (multidimensional) variable r, which can be chosen within a set of "feasible trades" R. There is a random variable

$\theta \in \Theta$ whose realization, occurring at date t_1, affects the agents' valuations of r. Trade only occurs once, at date $t_2 > t_1$. The trading problem we will consider is summarized by an allocation r^*, which associates an outcome r^*_θ with every possible realization of the random variable.[1] The two agents will be called B and S, for in most applications they will be viewed as a buyer (agent B) and a seller (agent S) of a given good; in this case the trade r will be decomposed as r = (p,q), with p being the price paid for q units of the traded good.

To implement the allocation r^*, at an initial date $t_0 < t_1$, the agents can sign a contract which specifies the conditions of trade between them in the future. The contract cannot be based on the realization of the random vsariable, which is observed by both agents but cannot be verified by any third party. The contract can however be based on observable and verifiable messages, as numerous as desired;[2] these messages can be sent at some dates between t_1 and t_2 and serve as a basis for an "announcement" game.

The timing of the problem is thus as presented in Figure 1:

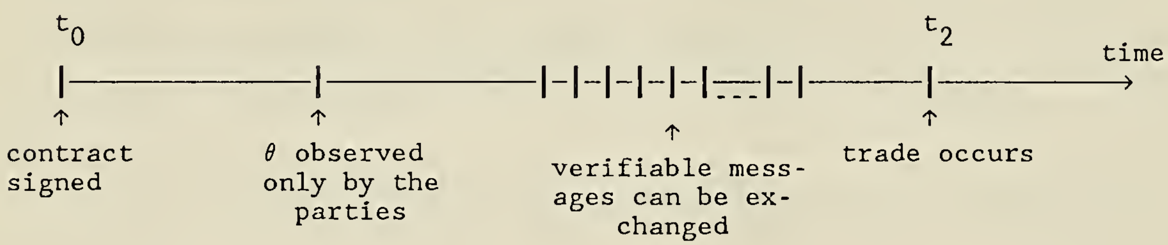

Figure 1

[1]In this simple framework, θ can serve as a parameter for describing the exogenous "state of nature"; this might no longer be the case when investment will be introduced.

[2]In the following, it will be sufficient to assume that for each agent the set of possible messages include Θ.

This is a standard implementation problem, as analyzed in particular by Maskin (1977, 1985) for the case of Nash implementation and by Moore and Repullo (1988) for the case of subgame perfect implementation: under weak conditions, mechanisms can be constructed that yield the desired outcomes as unique equilibria. These mechanisms may however rely on out-of-equilibrium *ex post* inefficient outcomes: Agents reveal truthfully the state of nature because they know that a deviation would lead to a suboptimal level of trade. In the absence of exogenous means of commitment, such a threat is not credible: through renegotiation, the parties would decide to move to an efficient level of trade instead at t_2. The next two subsections detail our assumptions on the *ex post* renegotiation process.

2. The Renegotiation Process

In this subsection, we present an axiomatic version of our assumptions on the *ex post* renegotiation process, as do Maskin-Moore (1988) and Green-Laffont (1988). In Subsection 3, we then detail an extensive form which generates these assumptions as endogenous predictions, and we stress the robustness of these predictions.

Three main assumptions will be made about renegotiation. The first one concerns its unavoidable character, while the other two focus on the ability of the contracting parties to monitor it.

Assumption 0: Renegotiation cannot be prevented, so that trade will be ex *post* efficient for all θ's.

Referring to Figure 1, this can be viewed as introducing a renegotiation stage taking place between the last exchange of verifiable messages and t_2, the time trade is to occur. Assumption θ says that the parties cannot

precommit at t_0 not to take advantage of renegotiation.

Assumption 1: A contract can specify a default option r that guarantees the parties at least the corresponding *ex post* utility level, $U_B(r,\theta)$ and $U_S(r,\theta)$.

This assumption means that the initial contract has to be voluntarily renegotiated, and protects each party against any unilateral violation of the agreement by the other party. The focus is thus only on Pareto-improving renegotiation.

Assumption 2: The initial contract can allocate the entire bargaining power in the *ex post* renegotiation stage to either of the parties.

Figure 2 visualizes our 3 assumptions about renegotiation. It is drawn for a particular state of nature θ, which defines a Pareto frontier in utilities space (U_S for the seller, U_B for the buyer). Assumption 0 says the outcome in state θ will lie on this frontier. Assumption 1 says that if the initial contract has r as default option (with corresponding utility levels $U_S(r,\theta)$ and $U_B(r,\theta)$ in θ, the outcome will lie between S and B on the Pareto frontier. Finally, Assumption 2 says that the initial contract can make sure the outcome is S (all bargaining power to the seller), or B (all bargaining power to the buyer). Of course, both the default option and the allocation of bargaining power can be made contingent on messages exchanged by the parties.

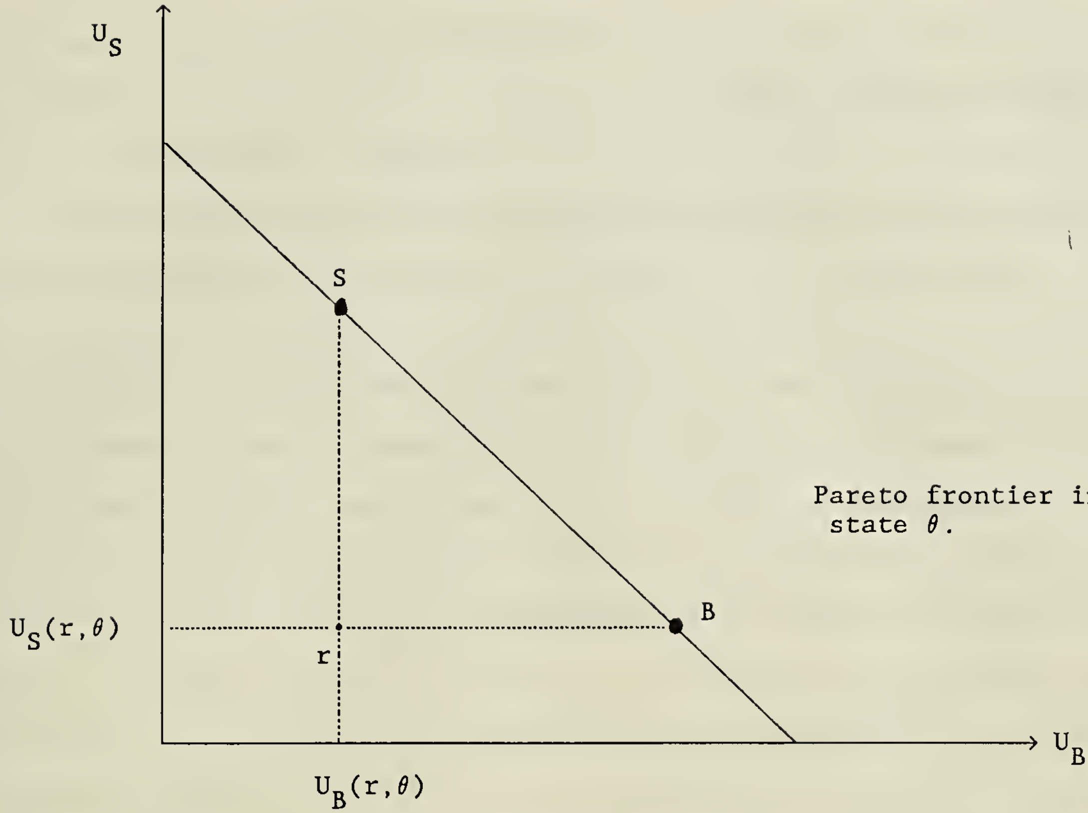

Figure 2

Our approach is thus to assume the existence of a "black box" that with each outcome associates an *ex post* efficient outcome (Ass. 0), but also the ability of the initial contract to influence this black box, through default options (Ass. 1) and extreme allocations of bargaining power (Ass. 2). In Section II to V, we show that this amount of renegotiation design is sufficient in many cases to overcome the problems created by the univerifiability of θ.

In terms of the existing literature, Assumption 0 is also made by Hart-Moore (1988), Green-Laffont (1988), and Maskin-Moore (1988). Assumption 1 is made by Green-Laffont and Maskin-Moore, but not by Hart-Moore. They instead assume trade to be voluntary: it takes place only if both parties agree to trade (in their framework, the quantity can be only 0 or 1), and courts cannot observe who refused to trade if it did not take place.

Consequently, a binding contract in their framework is a pair (p_0,p_1), i.e. a price schedule contingent on the level of trade (in Section III we will come back to the importance of this assumption). In comparison, Assumption 1 implicitly says that courts can observe whether each party has fulfilled its part of the default option (i.e. whether the buyer has paid p and the seller has supplied q).

Assumption 2 is <u>not</u> made by Green-Laffont or Maskin-Moore, who instead postulate an exogenous function $h(r,\theta)$ which selects an *ex post* efficient outcome which Pareto dominates r (i.e. an outcome on the Pareto frontier between S and B).[3] While they assume exogenous bargaining powers in the renegotiation stage, we allow the initial contract to monitor such powers partially, by giving all the power to one party.

Assumption 2 is partially made by Hart-Moore (in their extensive form, a high enough price differential p_1-p_0 means that the seller will have the power whenever renegotiation takes place, and a low enough price differential gives all the power to the buyer) but, as we will show, it is the <u>combination</u> of Assumptions 1 and 2 which provides the sufficient amount of renegotiation design which allows first-best outcomes to be implemented.

The trading problem we consider in this paper is quite simple. In a richer setting, the initial contract might design the renegotiation process in a variety of ways, as suggested in the conclusion. In our framework however, Assumptions 1 and 2 are sufficient to generate positive implementation results, so that more elaborate renegotiation design is not necessary. The following subsection provides a specific extensive form which satisfies these

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[3]For Maskin-Moore, one could however reinterpret $h(\cdot)$ as depending not only on r, but also on allocations of bargaining powers, so that our approach would be a special case of their general abstract implementation problem, for which they give general conditions of implementability. In this perspective, our work highlights the sensitivity of implementability to the choice of instruments on which $h(\cdot)$ can depend.

assumptions, and argues for their plausibility. Readers interested mainly in the implementation results derived from these assumptions may skip this subsection.

The idea of the subsection is to use a standard, infinite horizon sequential bargaining model with discounting to introduce Assumptions 0 and 1, and to show how Assumption 2 can emerge naturally in this context. Specifically, complete information and sequential bargaining without default options insures *ex post* efficiency (Rubinstein (1982)). The addition of default options then yields Assumption 1 (Binmore et.al. (1986)). One can then show that Assumption 2 can be supported by the introduction of "hostages", sums of money paid initially and to be refunded upon agreement. Alternatively, monitoring bargaining powers could be done by superimposing an additional, official sequential bargaining process upon the existing one, and making sure the official process would be the dominant one. In the Appendix, we show that the emergence of assumptions 0, 1 and 2 is quite robust to changes in the extensive form (order of the offers, moments at which the outside options can be imposed ...).

Note finally that we did not discuss *ex ante* renegotiation (i.e., renegotiation which would take place before the realization of the state of nature). This is not necessary here, since any (*ex ante*) Pareto-improvement could be included in the initial contract. This might no longer be the case, however, if (non verifiable) decisions were to take place at the beginning of the relationship. We will come back to this issue in Section V.

3. An Extensive Form

In keeping with most of the bargaining literature we model renegotiation as a sequential bargaining process. We assume a discrete, infinite-horizon model, with renegotiation rounds taking place at intervals of length Δ,

starting at t_0. For simplicity, assume there exists an integer n_2 such that $t_2=t_0+n_2\Delta$, and take the verifiable message stages between t_1 and t_2 not to coincide exactly with renegotiation rounds. Trade takes place only once, and can take place any date $t = t_2 + n\Delta$, with n any nonnegative integer. Delay is assumed to be costly (the discount factor is $\delta_i < 1$ for agent $i \in \{B,S\}$), so that *ex post* efficiency requires that trade takes place at t_2.

In this subsection, we model renegotiation as a standard alternating offer game (as in Rubinstein (1982)). In the Appendix, we show that our Proposition does not depend on this set of assumptions. Except for the end of this subsection, we concentrate on the subgames from t_2 on, for a given history of the game until $t_2-\Delta$. If one assumes away default option, each round one party can make an offer r, and the other party can either accept or reject and wait for Δ to make an offer. We know (Rubinstein (1982), Binmore (1982)) that the equilibrium of such a game is unique and *ex post* efficient, sot that assumption 0 is satisfied.

What if default options are introduced? The standard way of doing so is as shown in Figure 3, having the default option available to the player not making the offer.

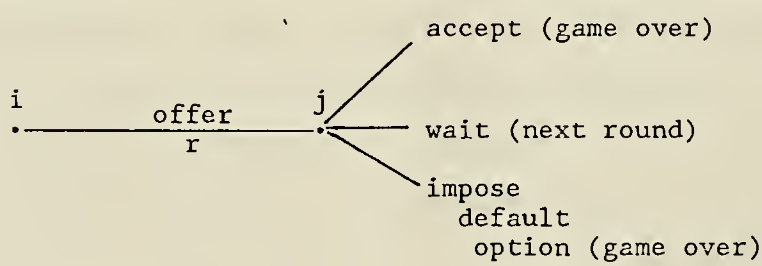

Figure 3

In round t, player i for example can make an offer, and the other player (j) can either impose the default option, accept the offer, or wait Δ in order

to make an offer. The game continues only in this last case. Binmore, Rubinstein and Wolinsky (1986) have shown that the outcome of such an infinite-horizon game with default option r is unique and as pictured in Figure 4. Call C the unique outcome of the game without default option, which lies on the Pareto-frontier. If it Pareto dominates r, it is also the outcome D of the game with default option; if instead, $U_S(C,\theta) < U_S(r,\theta)$ (respectively, $U_B(C,\theta) < U_B(r,\theta)$) then the outcome with default option is B (respectively, S).

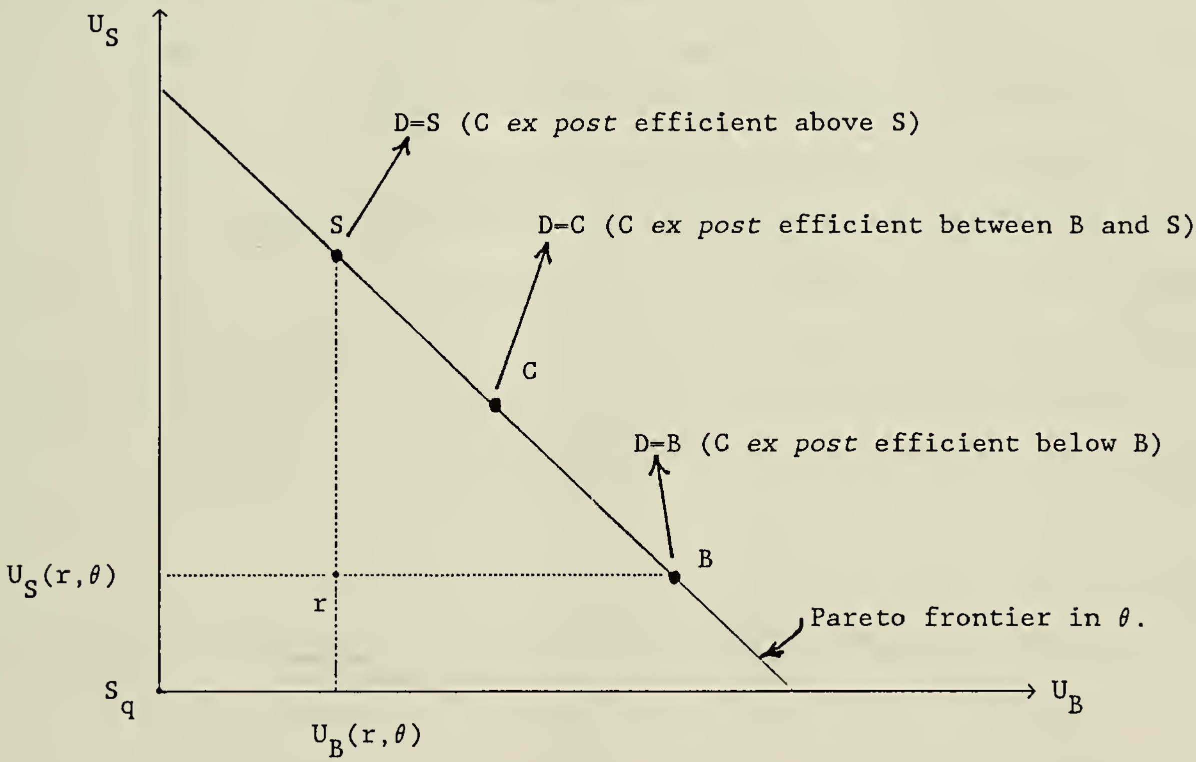

Figure 4

Two elements follow: first, Assumption 1 is now satisfied, on top of assumption 0, since the outcome is inbetween B and S; second, the default option is either irrelevant, or fully determines one party's payoff, that is the other party receives all the surplus from the implementation of C instead

of r.

While this takes care of default options, we still have to allow for distributions of bargaining powers. There are several ways to introduce this feature. One possibility, investigated in Aghion, Dewatripont and Rey (1988), is to add verifiable bargaining rounds to the above extensive form. These verifiable rounds will yield the desired result if they dominate the unverifiable ones, by being more frequent, or by involving "lasting" offers, that can be accepted and executed at any point in time.

Here, we want to privilege a less demanding interpretation, which relies simply on the existence of "hostages", represented by a sum of money T paid at t_2 by one party to the other and to be repaid, without interest, when trade occurs. Moreover, whenever the game ends (cf Figure 3), the hostage is repaid, but without interest, so that the party who has received it has enjoyed T from t_2 on, having to give it back only at $t' \geq t_2$, where t' is the time of agreement.

In Proposition 1, we show that such an extensive form from t_2 on yields Assumptions 0, 1 and 2, under the following assumption on preferences.

Assumption (I): $\forall i \neq j,\ i,j \in \{B,S\},\ \forall\ (p,q),\ \exists T \in \mathbb{R}$ s.t. $\forall \theta \in \Theta$,

$$U_i(T,0,\theta), \geq \underset{\tilde{p},\tilde{q}}{\text{Max}}\ U_i(\tilde{p},\tilde{q},\theta) \text{ s.t. } U_j(\tilde{p},\tilde{q},\theta) \geq U_j(p,q,\theta),$$

Assumption (I) says that high enough hostages, involving zero trade, can in any possible state give utility levels at least as high as the maximum utility level the party would achieve, having all the bargaining power and the other party being protected by a specific default option.

Assumption (I) thus means that, for any given default option (p,q), there exists T such that, for any θ, the situation is as depicted in Figure 5: the status quo at t_2 has moved to S'_q, which is more favorable than S, the *ex*

post efficient outcome which gives the buyer his default utility level. Assumption (I) would for example automatically be satisfied for utility functions such as $U_S = U_S(p-C(q,\theta))$ and $U_B = U_B(V(q,\theta)-p)$, as considered below, but may require unbounded utility w.r.t. money (e.g., if $U_S = W(p) - C(q,\theta)$).

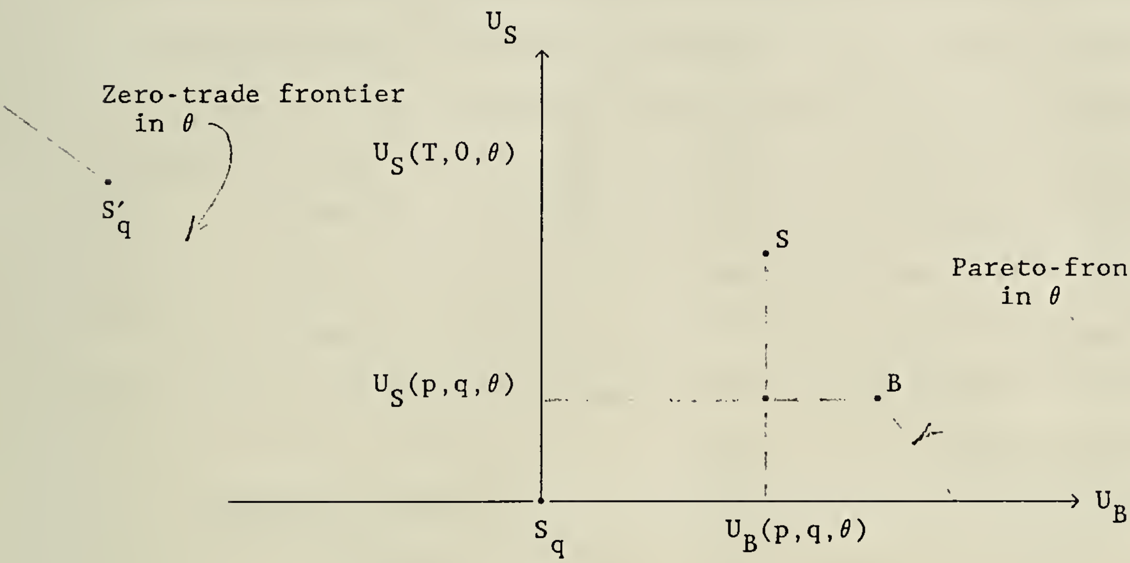

Figure 5

<u>Proposition 1</u>: Assume the game until t_2-Δ has yielded a default option (p,q) and Assumption (I) is satisfied. Then, there exists an appropriate transfer level such that the unique subgame-perfect equilibrium outcome from t_2 on is *ex post* efficient for all θ's and gives one party the utility associated to (p,q) in each θ.

<u>Proof</u>: Assume we want the buyer to receive $U_B(p,q,\theta)$ for every θ. By Assumption (I), there exists T s.t. the outcome is as in Figure 5 for every θ. This means that the outcome without default option and

status quo S'_q gives the seller more than the utility associated to S, since he could always refuse any agreement. Consequently, by the argument in Binmore, Rubinstein and Wolinsky (1986), the outcome with default option (p,q) is S. Q.E.D.

This proposition tells us that the existence of "hostages" allows one to generate assumptions 0, 1 and 2 in the standard bargaining framework. In the Appendix, we show that this generalizes to any stationary or nonstationary distribution of offers (instead of alternating offers), and to the possibility of imposing the default option just after having made a rejected offer. (Shaked (1987) shows this feature to be important in the context of the Binmore et.al. framework. Here, we can extend the result of Proposition 1 by allowing one party to give the other party a second default option in the last stage of verifiable messages.)

Let us now discuss the important assumption that, at any date from t_2 on, each party can enforce the current defaults option. A possible interpretation is that, when asked to do so by one of the agents, the court can (instantaneuosly) impose any trade associated with an (unrescinded) contract, i.e. either: (i) the court can then "decide" in place of the agents; or: (ii) the court can then fix arbitrarily constraining deadlines (so that the agents have no chance to rescind the contract before the deadline) and require very large penalties from any agent who does not choose the expected trade before a deadline. While the first assumption might not sound reasonable in most contexts, the second one deserves consideration, since deadlines are quite often observed, even if several reasons may argue for limiting their use (there may be some uncertainties associated with the production technology, or some unexpected events may arise, etc.), thereby decreasing the constraining capabilities of the court.

Another possible interpretation is the following; suppose that the court cannot force the agents to trade, but can only observe the agents' choices (this is a rather weak assumption, given the previous assumptions on technology; one may even suppose that the court observes the choices only after some delay, without altering the argument), and impose large (e.g., monetary) transfers in case of unilateral defection. Suppose moreover that, although the default option is not *ex post* efficient, each party can take an action which makes it *ex post* efficient given that action. This is quite natural for the seller, who could just decide to produce q. For the buyer, one could assume he has to combine the good produced by the seller with other inputs to make a product to be sold in turn; buying specific inputs may then make q *ex post* efficient. Then, in order to enforce the default option, the agent only has to take this specific action and threaten the other agent to alert the court: since no different trade is then mutually preferable, the other agent has no choice left but complying with the default option before the court can observe both choices.

In this formulation, although renegotiation is available, both agents are thus supposed to have access to some form of commitment and can potentially take irreversible decisions leading to inefficient outcomes.[4] This corresponds to the technological assumptions of Hart-Moore (1988)[5] and to natural interpretations of the analysis of Green-Laffont (1988) and Maskin-Moore (1988).[6]

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[4]Except in some trivial situations, it is impossible to design a contract such that all possible outcomes are ex-post efficient in all states of nature.

[5]There the agents have to choose, at a given - and unique - date, whether to push a button or not. Even if it is efficient, trade can feasibly only occur if both agents push the button at this date.

[6]Since renegotiated outcomes Pareto dominate initial outcomes, it is natural to interpret these initial outcomes as outside options that the parties could have imposed.

Finally, let us describe the game between t_1 and t_2. Parties can make renegotiation offers involving default options and allocations of bargaining power, as well as how to play the message stages which have not yet been reached. In each renegotiation round, the identity of the party making the offer is exogenously given. Since trade cannot yet occur for technological reasons, the other party can only accept or refuse the offer, then go to the next stage.

The crucial point is that, from t_1 on, the parties are playing some kind of generalized constant-sum game since, by Proposition 1, each subgame starting from t_2 which allocates all bargaining power to one party has a unique *ex post* efficient outcome. Therefore, if between t_1 and t_2 (possibly after some exchanges of messages) the parties agree on a default option and on the allocation of bargining powers, this agreement will not be renegotiated.

4. Preliminary Analysis

The actual game played by the agents of course depends on the initial contract. Moreover, given the renegotiation assumption, the natural equilibrium concept is (subgame) perfect equilibrium; accordingly, we will say that an initial contract implements an allocation $r = (r_\theta)_{\theta\in\Theta}$ if and only if all perfect equilibrium (expected) payoffs coincide with the payoffs associated with r.

We will focus here on a particular form on implementation. First, we will concentrate on contracts which, possibly contingent on messages sent by the parties, assign all bargaining power to one of the parties; we will denote by $\tilde{U}_i(\alpha,r,\theta)$ the agents utility level in the corresponding *ex post* efficient outcome, where $i\in\{B,S\}$, r is the default option and $\alpha\in\{B,S\}$ is the allocation of bargaining power. Second, we will require that at least one equilibrium of the game generated by the initial contract involves pure strategies only. It

should be stressed that allowing for a richer class of contracts, or for implementation in mixed strategies, would only strengthen the positive implementation results presented in the following sections.

We first state a proposition characterizing the implementation possiblities when mixed strategies are not allowed, and showing that attention can in this case be restricted to somewhat simple mechanisms. We then discuss a proposition, given in Appendix, which asserts that allocations which are implementable in this case are still implementable when mixed strategies are allowed. These two propositions rely on the *ex post* efficiency of the renegotiation process, and builds much on the work of Maskin and Moore (1988):[7]

<u>Proposition 2</u>: An *ex ante* allocation r^* is implementable in pure strategies (i.e., when mixed strategies are not allowed), using extreme bargaining powers, if and only if for any possible pair of states of nature (θ,θ') there exists a game $(\alpha_{\theta\theta'},r_{\theta\theta'})$ such that all $\alpha_{\theta\theta'}$'s $\in\{B,S\}$ and:

$$\tilde{U}_S(\alpha_{\theta\theta'},r_{\theta\theta'},\theta) \leq U_S(r^*_\theta,\theta), \tag{2.1}$$

$$\tilde{U}_B(\alpha_{\theta\theta'},r_{\theta\theta'},\theta') \leq U_B(r^*_\theta,\theta'), \tag{2.2}$$

The allocation can moreover be implemented using a one-stage direct mechanism which asks both agents to reveal the state of nature and assigns the game $(\alpha_{\theta\theta'},r_{\theta\theta'})$ to announcements (θ,θ').

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[7]The main differences are that here renegotiation is allowed between stages of the announcement game and that the (endogenous) renegotiation process which is applied after the announcement game can be contingent on the messages sent by the agents.

Let us provide some intuition for this proposition. Suppose first that an allocation r^* is implemented in pure strategies; consider the subgames starting after the realization of the state of nature, and denote by (b_θ, s_θ) the pair of equilibrium strategies associated with state of nature θ. Then for any (θ, θ'), the game à la Rubinstein associated with the pair of strategies $(b_\theta, s_{\theta'})$ clearly satisfies conditions (2.1) and (2.2), since by assumption , in state θ the seller prefers the game associated with (b_θ, s_θ) -which implements r^*_θ- to the game associated with $(b_\theta, s_{\theta'})$, and similarly in state θ' the buyer prefers $(b_{\theta'}, s_{\theta'})$ to $(b_\theta, s_{\theta'})$.

Conversely, once the state of nature is known, truthfully announcing the state of nature corresponds by construction to equilibrium strategies for the mechanism presented in the proposition. But the standard property of constant-sum games carries over to the present context: if only pure strategies are allowed, all equilibria have the same payoffs. Therefore, after the realization of the state of nature, the proposed mechanism implements the *ex post* efficient outcome r^*_θ. If the allocation r^* is also *ex ante* efficient, then this mechanism would not be renegotiated before the realization of the state of nature, and therefore would indeed implement this allocation.

Proposition 2′, presented in the appendix, provides a formal proof of the above proposition and extends it to the possibility of mixed strategies. It is shown that the introduction of mixed strategies can only help: any allocation which is implementable in pure strategies is still implementable when mixed strategies are allowed, using the same revelation mechanism as above, except that the state of nature is <u>sequentially</u> announced by the agents; on the other hand, some allocations might be implementable only in mixed strategies (i.e., conditions (2.1) and (2.2) are not necessary).

From now on, an (*ex ante*) allocation will thus said to be implementable if, for any states of nature θ and θ', there exists a game $(\alpha_{\theta\theta'}, r_{\theta\theta'})$ such that conditions (2.1) and (2.2) are satisfied. By *ex post* efficiency, they are respectively equivalent to:

$$\tilde{U}_B(\alpha_{\theta\theta'}, r_{\theta\theta'}, \theta) \geq U_B(r^*_\theta, \theta), \tag{2.1'}$$

$$\tilde{U}_S(\alpha_{\theta\theta'}, r_{\theta\theta'}, \theta') \geq U_S(r^*_\theta, \theta'), \tag{2.2'}$$

These two sets of conditions are also equivalent to:

$$\forall \theta \neq \theta', \; \tilde{U}_S(\alpha_{\theta\theta'}, r_{\theta\theta'}, \theta) \leq U_S(r^*_\theta, \theta) \leq \tilde{U}_S(\alpha_{\theta\theta'}, r_{\theta\theta'}, \theta'), \tag{2.3}$$

or:

$$\forall \theta \neq \theta', \; \tilde{U}_B(\alpha_{\theta\theta'}, r_{\theta\theta'}, \theta) \geq U_B(r^*_\theta, \theta) \geq \tilde{U}_B(\alpha_{\theta\theta'}, r_{\theta\theta'}, \theta'). \tag{2.3'}$$

These necessary and sufficient conditions call for two remarks:

First, no state θ'' different from θ and θ' interferes with states θ and θ'; this remark allows us to restrict our attention to the two-state case: an allocation r^* is implementable if and only if, considering any pair of states of nature θ and θ', the restriction of r^* to these two states would be implementable if θ and θ' were the only possible states of nature.[8]

Second, a pair of identical messages (θ, θ) never interferes with any other θ'; the associated game $(\alpha_{\theta\theta}, r_{\theta\theta})$ only appears in conditions which all together are equivalent to:

$$\tilde{U}_S(\alpha_{\theta\theta}, r_{\theta\theta}, \theta) = U_S(r^*_\theta, \theta) \tag{2.4}$$

There is thus no loss in generality in taking $r_{\theta\theta} = r^*_\theta$, which clearly satisfies this condition ($\alpha_{\theta\theta}$ can moreover in this case be arbitrarily chosen).

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[8]In terms of announcement games; the restriction of r^* might be ex ante inefficient if these two states were the only possible.

III. Risk sharing without investment.

This section addresses risk-sharing issues. Risk-aversion may affect the parties' objectives in various ways. The first subsection considers the natural case where both parties exhibit risk-aversion in their net payoffs (as in Hart-Moore [1988]). In this case, efficient risk-sharing involves co-insurance (both parties' payoffs increase with respect to the total surplus); we show how to implement this efficient risk-sharing using simple revelation schemes which assign no trade (q=0) as default option in the case of conflicting messages. In the following subsection, we analyze the more complicated case where both parties exhibit risk-aversion in their monetary revenues (as in Green-Laffont [1988]). In this case, there is no co-insurance with respect to the buyer's valuation (i.e., efficient risk-sharing implies that when this valuation increases, the buyer's net payoff must increase whereas the seller's net payoff must decrease). We show how the first best can still be implemented, using revelation schemes which involve either no trade or high trade as default options for *ex post* renegotiation. Our results are in sharp contrast with Green-Laffont [1988] where the choice of renegotiation rules is *a priori* restricted by an exogenous specification of bargaining powers.

1. The "co-insurance" case.

In this subsection we analyze the case where the objective functions of the seller and buyer are respectively defined by:

$$U_B(p,q,\beta) = u_B(v(q,\beta)-p) \tag{B1}$$

and

$$U_S(p,q,\sigma) = u_S(p-c(q,\sigma)), \tag{S1}$$

where p is the monetary transfer from the buyer to the seller and q the quantity produced by the seller at cost $c(q,\sigma)$; $\theta=(\sigma,\beta)$ is the state of nature and, given the preliminary analysis of section II.4, without loss of generality we can assume that there are only two states: $\theta_1=(\sigma_1,\beta_1)$ and $\theta_2=(\sigma_2,\beta_2)$; both u_B and u_S are increasing and concave. (Risk neutrality corresponds to the borderline case where $u''_B=u''_S=0$.) We shall also assume: $c_q>0$, $c_{qq}>0$, $v_q>0$, $v_{qq}<0$ and $\forall\beta,\sigma$, $c(\theta,\sigma)=v(q,\beta)=0$.

Let $((p^*_1,q^*_1),(p^*_2,q^*_2))$ be the first-best outcome, where (p^*_ℓ,q^*_ℓ) belongs to the Pareto-frontier in state θ_ℓ. This first-best allocation satisfies the following first-order conditions, for some Lagrange multiplier λ:

$$c_q(q^*_\ell,\sigma_\ell) = v_q(q^*_\ell,\beta_\ell),\ \ell = 1,2 \tag{3.1}$$

$$u'_S(p^*_\ell-c(q^*_\ell,\sigma_\ell)) = \lambda u'_B(v(q^*_\ell,\beta_\ell)-p^*_\ell), \tag{3.2}$$

(3.1) defines the *(ex-post)* efficient levels of trade q^*_ℓ independently from the sharing-rule agreed upon *ex-ante*, i.e., independently from the price structure p^*_ℓ. This price structure only enters (3.2), to achieve optimal risk-sharing. Furthermore, efficient risk-sharing implies co-insurance. Let us define for $\ell = 1,2$: $B^*_\ell = v(q^*_\ell,\beta_\ell)-p^*_\ell$, $S^*_\ell = p^*_\ell-c(q^*_\ell,\sigma_\ell)$, and $W^*_\ell = B^*_\ell+S^*_\ell$. Then from (3.2), $W^*_k<W^*_\ell$ if and only if $B^*_k\le B^*_\ell$ and $S^*_k\le S^*_\ell$, with at least one strict inequality. In other words, the two agents have the same ranking between states of nature in terms of the first-best outcomes in these states.[9]

We shall use this property in order to construct a simple mechanism G that implements the first-best outcome. Without loss of generality, assume

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[9] This "comonotonicity" property underlies Hart-Moore's Proposition 2(c).

$S_1^* \leq S_2^*$ and $B_1^* \leq B_2^*$. Consider the direct mechanism G defined by Figure 1:

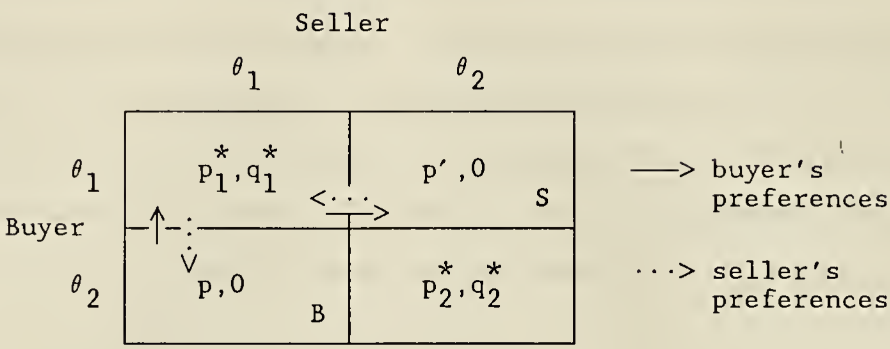

Figure 1

This mechanism specifies the first-best outcomes (p_1^*, q_1^*) and (p_2^*, q_2^*) on the diagonal and zero-output default options off the diagonal, with full buyer power below it and full seller power above it. There is no need to specify the bargaining power *ex-post* on the diagonal since the outcome (p_ℓ^*, q_ℓ^*) is relevant only in state ℓ, where it is efficient and therefore not renegotiated *ex-post*. For truthtelling by both agents to be a Nash equilibrium of the game associated with G, in state θ_1 we must have:

$$u_B(B_1^*) \geq \tilde{U}_B(B, (p, 0), \theta_1) \tag{3.3}$$

$$u_S(S_1^*) \geq \tilde{U}_S(S, (p', 0), \theta_1) \tag{3.4}$$

Since (p_1^*, q_1^*) is efficient in state θ_1, (3.3) is equivalent to:

$$u_S(S_1^*) \leq \tilde{U}_S(B, (p, 0), \theta_1) = u_S(p)$$

or:

$$S_1^* \leq p. \tag{3.5}$$

Similarly, inequality (3.4) is equivalent to:

$$B_1^* \leq -p' \tag{3.6}$$

In state θ_2, we must have:

$$u_B(B_2^*) \geq \tilde{U}_B(S,(p',0),\theta_2) \tag{3.7}$$

$$u_S(S_2^*) \geq \tilde{U}_S(B,(p,0),\theta_2), \tag{3.8}$$

which can be rewritten as:

$$B_2^* \geq -p' \tag{3.9}$$

$$S_2^* \geq p \tag{3.10}$$

It suffices to choose p such that $S_1^* \leq p \leq S_2^*$, and p′ such that $B_1^* \leq -p' \leq B_2^*$. Such a choice is always possible due to the <u>comonotonicity</u> between the S_ℓ^*'s and B_ℓ^*'s; hence:

<u>Proposition</u> <u>3</u>: In the coinsurance case, the first-best outcome can thus always be implemented through a direct mechanism G of the form described in Figure 1.

This mechanism is quite simple in that it specifies the same <u>zero</u> <u>output</u> level as default option for renegotiation above and below the diagonal. This relates to the coinsurance property, it will not hold in the general case where the objective functions U_B and U_S allow for "wealth effects": we will then have to consider more sophisticated mechanisms, where the starting points for renegotiation off the diagonal may involve a positive (<u>possibly</u> <u>high</u>)

level of output.

2. Allowing for wealth effects: the case of separable utility functions.

We consider the following utility functions which are separable in prices and quantitites:

$$U_B(p,q,\beta) = u(q,\beta)-v(p) \tag{B2}$$

$$U_S(p,q,\sigma) = w(p)-c(q,\sigma) \tag{S2}$$

where $\theta=(\sigma,\beta)$ is the state of nature (again, w.l.o.g. we shall assume two states of nature $\theta_1=(\sigma_1,\beta_1)$ and $\theta_2=(\sigma_2,\beta_2)$). Risk neutrality corresponds to the case where $v''=w''=0$. We shall assume that both the buyer and the seller are risk averse in terms of their monetary revenues: $v''>0$, $w''<0$. As before: $c_q>0$, $c_{qq}>0$, $u_q>0$, $u_{qq}<0$ and $\forall\beta,\sigma,c(0,\sigma)=v(0,\beta)=0$. Furthermore, we shall assume $u_\beta>0$, $u_{\beta q}>0$, $c_\sigma<0$, $c_{q\sigma}<0$ (that is, a higher β corresponds to a higher valuation for the buyer; and the higher the level of trade q, the more an increase of β affects the buyer's valuation. Similarly, a higher σ corresponds to a more efficient technique of production available to the seller, and the higher the q the bigger the cost reduction induced by an increase of σ).

a. Uncertainty on the seller's side.

We first analyze the case where all the uncertainty lies in the cost parameter σ, that is $\theta_1=(\sigma_1,\beta)$ and $\theta_2=(\sigma_2,\beta)$, and w.l.o.g. we assume $\sigma_1<\sigma_2$.

The first-best outcome $((p_1^*,q_1^*),(p_2^*,q_2^*))$ satisfies the following first-order conditions, for some Lagrange multiplier λ:

$$v'(p_1^*) = v'(p_2^*) = \lambda \tag{3.11}$$

$$u_q(q_\ell^*,\beta) = \lambda c_q(q_\ell^*,\sigma_\ell) \tag{3.12}$$

Equation (3.11) implies that $p_1^*=p_2^*=p^*$ since the buyer is risk-averse with respect to monetary payoffs (i.e., $v''>0$). On the other hand, equation (3.12), together with the assumptions $c_{q\sigma}<0$, $c_{qq}>0$, $u_{qq}<0$, implies that:

$$q_1^* < q_2^*. \tag{3.13}$$

The first-best outcome thus involves a level of trade q_2^* in state θ_2 which is greater than the level of trade q_1^* in state θ_1, where production is more costly ($\sigma_1<\sigma_2$). Unlike in section III.1, the ranking between state θ_1 and state θ_2 (in terms of the corresponding first-best outcomes (p_ℓ^*,q_ℓ^*)) is not necessarily the same for the buyer and the seller. The buyer will always rank state θ_2 above state θ_1 since in θ_2 he obtains a bigger quantity $q_2^* > q_1^*$ for the same price p^*; but for state θ_2 to be also preferred by the seller, it must be the case that:[10]

(S) $\qquad c(q_2^*,\sigma_2) < c(q_1^*,\sigma_1).$

In case this condition is met, we know from section III.1 that the first-best outcome can be implemented by the following simple mechanism (see Figure 2) which has zero trade as the default option and allocates full bargaining power to one party (seller or buyer).

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[10]Here we use our assumption that $c(0,\sigma)=0$. The case where there are fixed costs of production is analyzed in Aghion-Dewatripont-Rey [1988]: we show that the first-best outcome can still be implemented although through a non-extreme allocation of bargaining powers.

		Seller	
		σ_1	σ_2
Buyer	σ_1	(p^*, q_1^*)	$(p', 0)$ S
	σ_2	$(p, 0)$ B	(p_2^*, q_2^*)

Figure 2

Therefore, the above condition (S) is sufficient for the first-best to be implemented through simple renegotiation schemes. It turns out that (S) is also necessary for the first-best to be implemented through renegotiation design that allocates full bargaining power to the buyer or the seller. The reason is the following. Suppose there exists a revelation mechanism which implements the first-best allocation, and consider the negotiation process to be played when the buyer announces θ_2 and the seller announces θ_1. Whoever has the bargaining power, the seller's *ex-post* payoff associated with this renegotiation process necessarily increases when σ increases from σ_1 to σ_2:[11] the buyer's default utility level remains constant, and the total surplus increases. But the incentives constraint requires the seller to prefer (p^*, q_2^*) to this *ex-post* payoff in state θ_2, and this *ex-post* payoff to (p^*, q^*) in state θ_1.

[11]This property may not generalize to renegotiation processes in which no party has all the bargaining power. In a previous version (Aghion-Dewatripont-Rey [1988]), we provide an example where condition (S) is not satisfied and the first-best is however implemented, using a renegotiation process in which the two parties share the bargaining power.

b. <u>Uncertainty on the buyer's side</u>.

The choice of $q=0$ (no-trade) as the default option for renegotiation would remain appropriate as long as both contracting parties have a "comonotonic" ranking of the states of nature (in terms of the corresponding first-best outcomes). This comonotonicity property directly follows from co-insurance in section III.1, and is implied by condition (S) in the case of separable utility functions with uncertainty on the seller's side. It however turns out to be violated when introducing uncertainty on the buyer's side. As a consequence it may become necessary for the initial contract to specify default options which involve positive trade ($q>0$), hence the importance of our Assumption 1 introduced and discussed in Section II.

More precisely, let us consider the case where $\theta_1=(\sigma,\beta_1)$, $\theta_2=(\sigma,\beta_2)$ with $\beta_1<\beta_2$. The first-best outcome $((p_1^*,q_1^*),(p_2^*,q_2^*))$ will again involve full insurance in monetary terms, i.e.: $p_1^*=p_2^*=p^*$. Furthermore, the first-best level of trade will be higher in state θ_2 where the buyer's valuation for the good is bigger: $q_2^*>q_1^*$. However, the ranking between θ_1 and θ_2 cannot be the same for the buyer and the seller: the seller prefers θ_1 to θ_2 since in θ_1 it produces the lowest quantity q_1^* for the same price p^*; conversely, the buyer prefers θ_2 to θ_1 since he gets a higher quantity q_2^* for the same price p^* in state θ_2, which is furthermore the state where he values the good most.

Since comonotonicity is violated here, the above type of mechanism with no-trade as default option will not be appropriate to implement the first best: with such a mechanism the seller would have an incentive to lie in state θ_2 and take advantage of the fact that the buyer is willing to pay a lot more for moving away from no trade in that state. However, choosing a large q_0 as default option will implement the first-best outcome, provided that:

(B) $u_\beta(q,\beta)$ is unbounded above when $q \to +\infty$.

To prove our claim, let us consider the mechanism depicted on Figure 3.

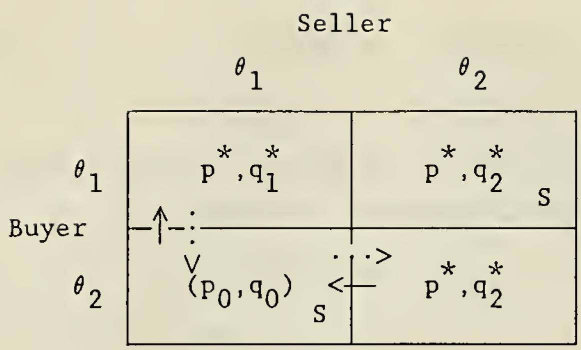

Figure 3

Off the diagonal this mechanism allocates full bargaining power to the seller,[12] and it specifies the outcome (p_0, q_0) as a default option for renegotiation below the diagonal. For the above mechanism the first-best solution (p^*, q_1^*), (p^*, q_2^*), it suffices that:

$$u(q_2^*, \beta_2) - u(q_0, \beta_2) \le v(p^*) - v(p_0) \le u(q_1^*, \beta_1) - u(q_0, \beta_1) \qquad (3.14)$$

Given that $\beta_2 > \beta_1$ and $q_2^* > q_1^*$, it suffices to choose q_0 sufficiently large and use the assumption that $u_\beta(q, \beta)$ is unbounded above. This assumption does indeed guarantee that for q large enough the LHS of (3.14) will become strictly less than the RHS. It then suffices to choose p_0 appropriately. Notice that no further assumption on U_B or U_S is needed.

The reason why choosing a higher level of trade as default option solves the implementation problem can be explained as follows. Let $\hat{q}_\ell$ (respectively, $\hat{p}_\ell$) be the final trade level (respectively, the final price) in state θ_ℓ =

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[12] Allocating full bargaining power to the buyer would not work in this case, since the seller would then always have an incentive to deviate in state θ_1: by doing so he would increase his profits from $p^* - c(q_2^*, \sigma)$ to $p^* - c(q^*, \sigma)$.

(σ,β_ℓ) after renegotiation under default option (p_0,q_0) and bargaining power to the seller. Since the buyer values the good more in state θ_2 than in state θ_1, we must have: $\hat{q}_1 < \hat{q}_2$ (see Figure 4).

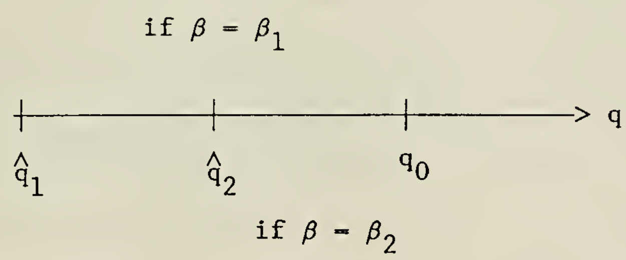

Figure 4

By choosing q_0 sufficiently large, we can at the same time prevent the buyer from lying in state θ_1 and the seller from lying in state θ_2. A deviation from truthtelling in state θ_1 will typically be unprofitable to the buyer if $\hat{p}_1$ is "large" and $\hat{q}_1$ is "low." Symmetrically a deviation from truthtelling in state θ_2 will be unprofitable to the seller if $\hat{p}_2$ is "low" and $\hat{q}_2$ is "large." We already know that $\hat{q}_1 < \hat{q}_2$. Now if we choose q_0 sufficiently large, we can also have $\hat{p}_1$ much larger than $\hat{p}_2$. The reason is that, even though $\hat{q}_2$ is larger than $\hat{q}_1$, decreasing the level of trade from a large quantity q_0 to $\hat{q}_2$ in state θ_2 will be more harmful to the buyer than decreasing the level of trade from q_0 to q_1 in state θ_1. ($\beta_2 > \beta_1$ and $V_{q\beta} > 0$). The final price $\hat{p}_\ell$ will then have to be much smaller in state θ_2 than in state θ_1 so as to guarantee the buyer his reservation utility level $u(q_0,\beta)-v(p_0)$ (which is moreover larger in state θ_2 than in state θ_1).

c. <u>Uncertainty on both sides</u>.

Consider the case where $\theta_1=(\sigma_1,\beta_1)$; $\theta_2=(\sigma_2,\beta_2)$, and where $\sigma_1\neq\sigma_2$, $\beta_1\neq\beta_2$. We distinguish in fact two subcases. The first one is defined by $\theta_1=(\sigma_1,\beta_1)$;

$\theta_2=(\sigma_2,\beta_2)$, with $\beta_2>\beta_1$ and $\sigma_2>\sigma_1$, that is, state 2 is more favorable both in terms of buyer valuation and of seller's cost. In that case, the first-best $(p^*,q_1),(p^*,q_2)$ can be implemented through the same mechanism as in (b). (The starting point q_0 may have to be chosen even larger than in (b), since the efficient quantity increases more between the two states θ_1 and θ_2.)

The second subcase is defined by $\theta_1=(\beta_1,\sigma_2)$; $\theta_2=(\beta_2,\sigma_1)$, with $\beta_2>\beta_1$ and $\sigma_2>\sigma_1$, that is, state has higher buyer valuation, but also higher seller cost.

Let $q_1^*=q^*_{\beta_1\sigma_2}$ and $q_2^*=q^*_{\beta_2\sigma_1}$ be the corresponding first-best quantity levels. Then, either $q_2^*>q_1^*$ or $q_2^*<q_1^*$ (the case $q_2^*=q_1^*$ is trivial, since we also have $p_2^*=p_1^*$). If $q_2^*>q_1^*$, we can implement the first-best $(p^*,q_1^*),(p^*,q_2^*)$ through the same mechanism as in (b), using again the assumption that the function u is unbounded. If $q_1^*>q_2^*$ the mechanism used in (b) will not work: with full seller power above the diagonal and a default option $r_{12}=(p^*,q_2^*)$, the seller would always have an incentive to lie in state θ_1, and thereby produce a smaller quantity $(q_2^*<q_1^*)$ for the same price p^*. We can implement the first best, however, through the mechanism depicted in Figure 5, which allocates full bargaining power to the buyer:

		Seller θ_1	Seller θ_2
Buyer	θ_1	p^*,q_2^*	p^*,q_1^* B
	θ_2	$p,0$ B	p^*,q_1^*

Figure 5

A sufficient condition for implementation is then:

$$c(q_2^*,\sigma_1) > c(q_1^*,\sigma_2) \tag{3.15}$$

It turns out that (3.15) is implied by the following conditions:

$$(\bar{S}): \quad c(q^*_{\sigma_1\beta_1},\sigma_1) > c(q^*_{\sigma_2\beta_1},\sigma_2) \text{ and } c(q^*_{\sigma_1\beta_2},\sigma_1) > c(q^*_{\sigma_2\beta_2},\sigma_2),$$

which generalize the above condition (S) to the case of two-sided uncertainty. Indeed, (3.15) can be rewritten: $c(q^*_{\sigma_1\beta_2},\sigma_1)>c(q^*_{\sigma_2\beta_1},\sigma_2)$. But the first order conditions (3.12) implies that:

$$q^*_{\sigma_1\beta_1} < q^*_{\sigma_2\beta_1} < q^*_{\sigma_2\beta_2}; \tag{3.16}$$

and thus (3.15) results immediately from ($\bar{S}$), (3.16), and the fact that $c_q>0$. We have thus proven the following proposition:

<u>Proposition</u> <u>4</u>: When the utility functions of the buyer and the seller are defined by (B2) and (S2) then, provided the $u_\beta(q,\beta)$ is unbounded when $q \to +\infty$, a sufficient <u>condition</u> for the first-best outcome (p^*_ℓ,q^*_ℓ), $\ell = 1,2$ to be implemented is:

$$(\bar{S}): \quad \forall\beta = \beta_1,\beta_2,\ c(q^*_{\sigma_1\beta},\sigma_1) > c(q^*_{\sigma_2\beta},\sigma_2),$$

where $q^*_{\sigma\beta}$ is the first best quantity in state $\theta=(\sigma,\beta)$.

IV. <u>INVESTMENT</u> <u>UNDER</u> <u>RISK</u> <u>NEUTRALITY</u>

We now switch our focus from risk sharing to investment. A number of authors (Klein, Crawford and Alchian (1978), Williamson (1979) and, more recently, Grout (1984), Tirole (1986) and Grossman and Hart (1986)) emphasize that the fear of *ex post* breach or renegotiation of the terms of a relationship may induce underinvestment in assets which are specific to this

relationship. Hart and Moore (1988) and Green and Laffont (1988) have provided an analysis of this problem in an optimal-but-incomplete contract setting with risk-neutral agents. In this Section and the following one, using the same framework as before, we show how renegotiation design can be useful in inducing the right incentives to invest. In this section agents are assumed to be risk-neutral; efficient investment decisions and levels of trade can then be achieved with a somewhat simple contract, which does not require any exchange of messages: the agents only agree initially on a default option (price, quantity) and renegotiate ex *post* giving all the bargaining power to one party. The next section discusses how this efficiency result can be extended to the case when one or both parties are risk-averse.

Let us first introduce investment in our framework. Investment will be assumed to be specific to the relationship: the associated costs have to be sunk before the realization of the state of nature, and the level of investment only affects the parties' valuation for their specific trade.

As in the Hart-Moore framework, assuming risk-neutrality and ruling out direct externalities, the objective functions of the seller and buyer now become respectively:

$$U_S(p,q,i,\theta) = p - c(q,i,\sigma) - \varphi(i) \tag{4.1}$$

$$U_B(p,q,i,\theta) = v(q,j,\beta) - p - \psi(j) \tag{4.2}$$

The investment cost functions are assumed to be convex (φ' and $\psi' > 0$, φ'' and $\psi'' > 0$), and investment is assumed to raise the amount of quasi-rents from the relationship ($v_j > 0$, $c_j > 0$, but $v_{jj} < 0$ and $c_{jj} > 0$), the more so the higher the amount traded ($v_{qj} > 0$, $c_{qj} < 0$).

The goal of the parties is to design a contract to achieve the first-best allocation. Assuming a finite number of states of nature,

$l=1,\ldots,N$, and denoting the probability of state θ_ℓ by Π_ℓ (assumed w.l.o.g. to be positive), the first-best allocation is a pair of investment levels (i^*,j^*) and a vector of trade levels $q^* = (q_1^*,\ldots,q_\ell^*,\ldots,q_N^*)$ such that:

$$c_q(q_\ell^*,i^*,\sigma_\ell) = v_q(q_\ell^*,j^*,\beta_\ell) \quad \forall \ell = 1,\ldots,N \tag{4.3}$$

$$\sum_\ell \pi_\ell \, c_i(q_\ell^*,i^*,\sigma_\ell) + \varphi'(i^*) = 0 \tag{4.4}$$

$$\sum_\ell \pi_\ell \, v_j(q_\ell^*,j^*,\beta_\ell) - \psi'(j^*) = 0 \tag{4.5}$$

Condition (4.3) characterizes for each state θ_ℓ the *ex post* efficient level of trade q_ℓ^*, given (i^*,j^*), whereas (4.4) and (4.5) characterize the *ex ante* efficient investment levels.

Bilateral risk neutrality makes the price structure irrelevant: only the average price matters, which determines how the surplus is divided between the parties. Conversely, for given investment choices, the state of nature uniquely determines the *ex post* efficient output level, which will therefore be achieved *ex post* through renegotiation, whatever the starting point of the bargaining process and the relative bargaining power of the parties. Thus, if the parties choose efficient levels of investment, renegotiation by itself guarantees the implementation of the first-best allocation.[13]

The agents' problem thus becomes: Does there exist a contract which induces (i^*,j^*)? The answer to this question is here surprisingly simple:

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[13] More precisely, it guarantees that given the investment levels, the *ex ante* surplus (considered at some date before the realization of the state of nature), will be equal to the optimal one. This surplus can then be correctly divided through adequate lump sum transfers.

Proposition 5:[14] The following contract implements the first best outcome: Give all renegotiation power to the seller and choose a starting point (p,q), where q is defined by

$$\sum_{\ell} \pi_{\ell} v_j(q,j^*,\beta_{\ell}) = \psi'(j^*),$$

and p correctly divides the efficient surplus, according to *ex ante* individual rationality.

Proof: In this mechanism, the seller is made residual claimant. He will thus choose $i=i^*$, provided the buyer chooses $j = j^*$. But the buyer, without bargaining power, knows that his payoff will be $v(q,j,\beta_{\ell})-p$ in state θ_{ℓ} for investment j. By the definition of q, $j=j^*$ is thus a dominant choice for the buyer (remember that $v_{jj} < 0$, and $\psi'' > 0$).

Q.E.D.

This mechanism is quite simple, since the starting point of the bargaining process is not contingent on any messages exchanged by the agents. One party has all the bargaining power, and is thus made residual claimant, whereas the starting point is determined so that the other party has in the average the correct incentives to invest.

In our framework, avoiding underinvestment is thus possible, which is in sharp contrast with the incomplete contract literature on this topic. Let us first compare our framework with that of Hart and Moore. Theirs differs from ours in several respects. First, of course, they do not allow for renegotiation design as freely as we do. As discussed in earlier Sections, their unofficial offers "have the last word", so that the starting point of bargaining (a pair of prices (p_1,p_0), contingent on trade or no trade)

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[14] After having written the previous version of this paper, it happened to us that Chung [1988] has independently provided a similar result.

completely determines the relative bargaining power of the parties given $(i, j, \beta_\ell, \sigma_\ell)$. Second, trade is a zero-one decision, instead of our continuous output level. Third, and this is the key to underinvestment, their assumptions on the enforcement technology imply that renegotiation, if any, takes always place from a no-trade default option.[15] This introduces a downward bias in the distribution of default options, which disappears only when a pair (p_1, p_0) can be found such that it is renegotiation-proof whatever i and j, or when one party's investment is irrelevant (so that the other one can be made residual claimant).

In contrast, our scheme works because the default option can be set at an "average" trade level. The key difference is thus the ability of the court to verify that each party has done his part of the trade (i.e. the seller provided q and the buyer paid p).

Hart and Moore are in keeping with the literature on underinvestment, where the key factor is the threat of no trade, whether bargaining is not modeled explicitly, modeled cooperatively, or non cooperatively.[16] In that case, renegotiation is such that for each state of nature both payoffs (weakly) increase with the total surplus; therefore, no expected return from an increase in investment can be higher than the corresponding expected increase in surplus and moreover the expected return is smaller for at least for one party, hence the underinvestment effect. Once the default option can be freely chosen, the return from an increase in investment can in some states

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[15] The combined assumptions that "it takes two to trade" and that the court cannot observe who refused to trade, imply that bargaining occurs only when trade is efficient and would not take place at initial prices, in which case it starts from $(p=p_0, q=0)$. Bargaining thus never starts from $(p=p_1, q=1)$.

[16] For example, Grout (1984) assumes generalized Nash bargaining, and Tirole (1986) investigates several bargaining mechanisms. In both cases, the no-trade outcome is the inefficient default outcome.

of nature be higher than the corresponding increase in surplus; indeed, if one party has no bargaining power, his payoff increases less than the surplus if, as before, the default level of trade is lower than the efficient one, but it increases more than the surplus in the converse case: when the default trade q is higher than the efficient level of trade q(i,j), the increase in the default utility level (measured by $v_j(q,j,\beta)$ or $c_i(q,i,\sigma)$) is higher than the increase in surplus (measured by $v_j(q(i,j),j,\beta)$ or $c_i(q(i,j),i,\sigma)$, using the envelop theorem).[17] In our mechanism the buyer has no power whatsoever in the bargaining game. With a no-trade default option, j=0 would be optimal, because cutting j involves no loss of quasi-rents at all for the buyer. Instead, for this "average" q, the buyer is *ex ante* "locally" residual claimant: by cutting j, the buyer's loss of utility in terms of outside option (p,q) equals the loss in total expected quasi-rents, so that $j=j^*$ is optimal.

Since choosing a higher default level of trade would give the buyer incentives to overinvest, it is possible to deal with one-sided direct externalities as well. Let us for example replace (4.1) by:

$$U_S = p_\ell - c(q_\ell, i, j, \sigma_\ell) - \varphi(i) \tag{4.1'}$$

with $c_j < 0$, $c_{jj} > 0$. In the first-best outcome, j would have to be introduced in the cost components of (4.3) and (4.4), while (4.5) would become:

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[17] In the framework of Hart and Moore, depending on the realization of the state of nature and on the price differential, one of the parties has all the bargaining power. Since trade is assumed to be voluntary, the parties have therefore for each state of nature either the correct incentives to invest, or no incentives at all. If "no trade" was assumed to be voluntary (i.e., it "takes two for no trade"), then for each state of nature both parties would have either the correct incentives, or too high incentives to invest; this would then lead to an overinvestment effect.

$$\sum_{\ell} \pi_{\ell} \left(v_j(q_{\ell}^*, j^*, \beta_{\ell}) - c_j(q_{\ell}^*, i^*, j^*, \sigma_{\ell})\right) = \psi'(j^*) \qquad (4.5')$$

The optimal mechanism would remain exactly as in Proposition 5, but with j^* redefined as above. The impact of this direct externality on the contract is thus simply to raise the level of trade associated with the default option.

Let us close this Section by noting that Green and Laffont (1988) do allow, like us, the parties to freely choose starting points of the bargaining process. They however introduce a separability assumption between investment and the level of trade ($v_{qj} = c_{qi} = 0$), so that the choice of starting trade level does not affect incentives to invest. The reason why the possibility of renegotiation leads to underinvestment only comes from the threat of disagreement, which provides a stronger bargaining position for the buyer the higher the seller's investment (relative bargaining powers are moreover exogenous). In this sense, while granting the parties the ability to freely choose prices and quantities as starting points, Green and Laffont exogenously place themselves in the framework of the underinvestment literature.

V. INVESTMENT AND RISK AVERSION

Let us now introduce risk-sharing issues as an additional concern. Is it still possible to implement the first-best allocation ? A natural idea is to adapt the previous mechanism to this new situation. Let us denote by $U_B(q,p,j,\beta)$ and $U_S(q,p,i,\sigma)$ the buyer's and seller's VNM utility functions; suppose for instance that there exist a quantity q and a price p such that:

$$E_{\beta}[U_{Bj}(q,p,j^*,\beta)] = 0 \qquad (5.1)$$

$$E_{\beta}[U_B(q,p,j^*,\beta)] = U_B^* \tag{5.2}$$

and suppose that the agents agree on using this pair (p,q) as a default option and on giving all the bargaining power to the seller. If this agreement is not renegotiated, the buyer's expected utility level associated with investment j is given by $E_{\beta}[U_B(q,p,j,\beta)]$. Therefore condition (5.1), combined with a concavity assumption of the objective function with respect to the level of investment, guarantees that the buyer will choose the efficient level of investment j^*. Moreover, condition (5.2) asserts that the buyer's expected level of utility will be equal to its first-best.

This mechanism, however, is based only on "average" considerations, and is thus likely to induce inefficient risk-sharing; it would therefore be subject to renegotiation before the realization of the state of nature. This leads us to be more explicit on renegotiation at that stage (which we will refer to as *interim* renegotiation).

1. Interim renegotiation design

A possible way is to introduce a renegotiation process at some point between the investment decisions and the realization of the state of nature. What assumptions can we make on this *interim* renegotiation process ? In particular, is it reasonable to maintain the assumptions 0, 1 and 2 for this renegotiation stage, as we did for *ex post* renegotiation ? Let us discuss these assumptions in turn.

Hypothesis 0 asserts that efficiency is achieved through renegotiation. It seemed natural for *ex post* renegotiation, since in that case both agents

know the efficient outcomes and bargain directly on these outcomes. In the case of *interim* renegotiation, the issue is an allocation which specifies an outcome for each possible state of nature. The parties know the efficient allocations; however, they do not bargain directly on allocations, but rather on mechanisms which *in fine* will implement some allocations. If all efficient allocations can be implemented in this way (as in the situations analyzed in Section III), assumption 0 seems as reasonable for *interim* renegotiation as for *ex post* renegotiation. Otherwise, the natural assumption is that the issue of *interim* renegotiation is efficient in the class of implementable allocations. Remember that this assumption represents a constraint on the possible contracts, so that relaxing it could only enlarge the class of situations where the first-best can be implemented.

Hypothesis 1 asserts that if the parties initially agree on some mechanism (which may lead to inefficient risk-sharing, particularly if one party shirks on the investment decision) then, in terms of expected level of utility, no party can loose from *interim* renegotiation. This assumption, generally considered as a minimal property of voluntary renegotiation, will be maintained in the following.

Hypothesis 2, which supposes that the initial contract can give all the bargaining power to one of the parties, is more problematic for *interim* renegotiation. In particular, providing some foundations as we did in Section II.3 for *ex post* renegotiation may be more difficult.[18] In fact, in the same lines as in the previous analysis, the ability of concentrating the bargaining power on one side is indeed very powerful. More precisely, combined with assumptions 0 and 1, it enables the agents to implement the first-best outcome

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[18] In particular, the cost of delaying agreement is not as clear as when trade is at stake. Also, the possibility of (unilaterally) imposing an inefficient risk-sharing rule, which could not be renegotiated before the realization of the state of nature, may be more questionable; conversely, the absence of irreversibility makes it difficult to set up "hostage" mechanisms in the spirit of those used in section II.3.

using the simple mechanism described above:

<u>Proposition</u> <u>6</u>: Assume that assumptions 0, 1 and 2 hold for <u>both</u> *interim* and *ex post* renegotiation, and that there exist a pair of price and quantity (p,q) satisfying conditions (5.1) and (5.2).[19] Assume moreover that the buyer's utility function $U_B(q,p,j,\beta)$ is strictly concave with respect to the investment level j. Then the following contract implements *the first best outcome:* give all renegotiation power to the seller, both for *interim* and *ex post* renegotiation, and choose this pair (p,q) as the unique default option after the realization of the state of nature.

<u>Proof</u>: If this agreement is not renegotiated before the realization of the state of nature, the buyer's expected utility level associated with investment j is given by $E_\beta[U_B(q,p,j,\beta)]$. Since the buyer has no bargaining power before the realization of the state of nature, this also represents his expected utility level after renegotiation; the efficient level of investment, j^*, is therefore a dominant choice for the buyer, whatever the investment chosen by the seller. Suppose now that the seller chooses some investment i, and consider the outcome of the *interim* renegotiation: it is efficient by assumption 0 and gives U_B^* (whatever is i) to the buyer. Therefore the seller is *ex ante* residual claimant and will thus choose $i=i^*$, since the buyer chooses $j = j^*$.

Q.E.D.

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[19] In the absence of a separability assumption of the objective functions between prices and quantities, such a pair (p,q) may not exist. It is however likely to exist if for instance the objectives are given by a not too much concave function of the profits.

Proposition 6 relies on the ability to give all the bargaining power to one party at the *interim* renegotiation stage, but does not require to give the power sometimes to one party and sometimes to the other. It can thus be extended to situations where, although assumption 2 is not satisfied for *interim* renegotiation, one party has naturally all the bargaining power at this stage. Suppose for instance that the *interim* renegotiation game is defined by an extensive form such as in Section II.3 (i.e., renegotiation consists of sequential rounds where one party proposes a new mechanism to be played after θ, which the other party accepts or refuses to switch to). Since there is no cost of delay, one party (namely, the last to make an offer before the state of nature is realized) has *de facto* all the bargaining power, and therefore, although the initial contract may not impose <u>who</u> has the bargaining power, proposition 6 is valid (reversing the role of the parties, if necessary, in the proposed mechanism).

2. <u>No</u> <u>interim</u> <u>renegotiation</u> <u>design</u>

The previous analysis rests on one party's having all the bargaining power for *interim* renegotiation. Since assumption 2 may be questionable for this stage of renegotiation, and since no party may have "spontaneously" all the *interim* bargaining power, it is interesting to see what can be done such situations. We have studied the case when, as in section III.1, the objectives of the two parties depend on their net revenues, i.e. the agents' VNM utility functions are given by:

$$U_S(p-c(q,i,\sigma)-\varphi(i)) \tag{5.3}$$

and

$$U_B(v(q,j,\beta)-p-\psi(j)) \tag{5.4}$$

with $U'_S, U'_B > 0$, $U''_S, U''_B < 0$, and the same usual conditions as before on the cost and valuation functions c and v. In this case, provided assumption 1 is valid, some mechanisms can indeed implement the first-best allocation, <u>whatever the parties' relative bargaining power at the interim renegotiation stage</u>. We shall briefly discuss at the end of this Section how the introduction of wealth effects would complicate the analysis.

In order to acquire some intuition on the design of these mechanisms, which have their own interest, it will be convenient to first address the case of unilateral risk aversion. But let us first briefly describe the first-best allocation in the general case; the first-order conditions of the problem yield, for some Lagrange multiplier λ:

$$U'_S(p^*_\ell - c(q^*_\ell, i^*, \alpha_\ell) - \varphi(i^*)) = \lambda U'_B(v(q^*_\ell, j^*, \beta_\ell) - p^*_\ell - \psi(j^*)) \quad \forall \ell \tag{5.4}$$

$$c_q(q^*_\ell, i^*, \sigma_\ell) = v_q(q^*_\ell, j^*, \beta_\ell) \qquad \forall \ell \tag{5.5}$$

$$\sum_\ell \pi_\ell \, U_S(p^*_\ell - c(q^*_\ell, i^*, \alpha_\ell) - \varphi(i^*)) \; [c_i(q^*_\ell, i^*, \sigma_\ell) + \varphi'(i^*)] = 0 \tag{5.6}$$

$$\sum_\ell \pi_\ell \, U_B(v(q^*_\ell, j^*, \beta_\ell) - p^*_\ell - \psi(j^*)) \; [v_j(q^*_\ell, j^*, \beta_\ell) - \psi'(j^*)] = 0 \tag{5.7}$$

The last three conditions are identical to (4.3)-(4.5). In particular, the investment levels and the state of nature completely still determine the *ex post* efficient output level (condition (5.5)), and *ex post* renegotiation will still again yield the *ex post* efficient surplus. But because of risk-aversion, the price structure matters (it determines the relatives shares of surplus in each state of nature, and is characterized by condition (5.4)), so that *ex post* renegotiation does no longer guarantee *interim* efficiency.

a. Unilateral risk aversion

Assume for example the seller to be risk neutral (the *analysis* would be symmetric under buyer risk neutrality). In this case, optimal risk-sharing requires the buyer's utility to be constant across the states of nature: $v(q^*_\ell, j^*, \beta_\ell) - p^*_\ell = B^*$. The mechanism of Section IV would not achieve that since, at least for a positive q, $v(q, j^*, \beta_\ell)$ is not constant across states of nature. If we assume $v(0,j,\beta)=0$ (zero buyer valuation at q=0, whatever the prior investment in the relationship and the state of nature), a unique starting point $(p_0, 0)$ with (*ex post*) full seller power, would provide the buyer with a constant payoff across states of nature. Choosing $p_0=-B^*$ would set this payoff at the first-best level. This mechanism would however not induce $j=j^*$, since the buyer would reap none of the surplus generated by his investment.

One could however introduce the threat of another starting point in case of buyer underinvestment. The next proposition illustrates this idea:

Proposition 7:[20] Call $\underline{\beta}$ the lowest β. If $v(0,j,\beta)=0$ for any j and any β, and if $v_j(q,j,\beta)$ is unbounded when q increases, then there exists a quantity q′ such that the following contract implements the first best for any *interim* renegotiation process satisfying assumption 1: after the realization of the state of nature, give all the *ex post* bargaining power to the seller, and let him choose between $(p_0, 0)$ and (p', q') as starting points, where p_0 and p' are given by $v(q', j^*, \underline{\beta}) - p' = -p_0 = -B^*$.

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[20] This proposition and the following one rely on having a finite number of states of nature, or more precisely on having bounded values for β and σ, and positive probabilities for the extreme values. If it is not the case, it may be possible to use a similar kind of contracts as those used here to implement "almost first-best" allocations; this however deserves further research.

Proof: *Ex post* renegotiation always yields *ex post* efficiency. Suppose $j=j^*$, and consider the seller's investment decision. If the scheme is not renegotiated at the *interim* stage, then for any investment level the seller *ex post* strictly prefers $(p_0,0)$ as starting point when $\beta_\ell>\underline{\beta}$ (since $v_\beta > 0$ for $q > 0$), and is indifferent between $(p_0,0)$ and (p',q') for $\beta_\ell=\underline{\beta}$: the buyer therefore receives a stable payoff across states of nature, which means that the proposed scheme yields not only *ex post*, but also *interim* efficiency; moreover, the buyer exactly obtains $U_B(B^*)$, which is not affected by the seller's investment. Thus if $j=j^*$ the seller is (*ex ante*) residual claimant and chooses $i=i^*$, leading to the first best allocation.

Suppose $j>j^*$. Then *ex post* the seller strictly prefers $(p_0,0)$ as starting point ($v_j>0$ for $q>0$, and $v_j=0$ for $q=0$). This is *interim* efficient, and thus would not be renegotiated before θ; but compared to $j=j^*$, the buyer does not receive any additional quasi-rents from his investment, and therefore prefers j^*.

Suppose $j<j^*$. Since v_j is unbounded when q increases, for all $j \in [0\ j^*[$, one can define $\tilde{q}(j)$ such that $\tilde{\pi}(v(\tilde{q}(j),j^*,\underline{\beta})-v(\tilde{q}(j),j,\underline{\beta})) = \psi(j^*) - \psi(j)$. This defines a continuous function $\tilde{q}(j)$ on $[0\ j^*[$ which moreover has a finite limit at j^*. Call its supremum on this interval $\tilde{q}$, and choose $q' > \tilde{q}$. When $\beta_\ell=\underline{\beta}$, the seller wants then to start from (p',q'). This may not yield optimal risk-sharing but, denoting by $\tilde{\pi}$ the sum of all π_ℓ's such that θ_ℓ involves $\underline{\beta}$ as buyer's valuation, through *interim* renegotiation the buyer's expected utility cannot be higher than $U_B[\tilde{\pi}(v(q',j,\beta_\ell)-p')+(1-\tilde{\pi})(-p_0)-\psi(j)]$, which corresponds to the situation where the buyer has all the *interim* bargaining power (things could become even worse for the buyer for low j's, as the seller would begin choosing (p',q') as starting point in states where $\beta_\ell>\underline{\beta}$ too). By choosing $q'>\tilde{q}$, the expected loss in

terms of quasi-rents in states where $\beta_\ell = \underline{\beta}$ suffices, by the definition of $\tilde{q}$, to more than compensate the cut in $\psi(j)$; therefore the buyer again prefers j^*.

Q.E.D.

As in the previous Section, this mechanism relies on the ability of enforcing a given level of trade, which is used as a threat to deter underinvestment. If the buyer chooses $j=j^*$, zero trade is used as the unique starting point, which stabilizes across the sates of nature ($B_\ell = B^*$); the scheme is thus *interim* efficient, and moreover the buyer's expected utility is not affected by the seller's investment, which in turn gives the seller the right incentives to invest. Zero trade is also used if $j>j^*$, which prevents the buyer to overinvest. For $j<j^*$, the seller prefers in some states of nature to start bargaining from a very high trade level, which is quite detrimental to the buyer, since each omitted unit of investment much lowers the value of his default option.

Once again, (i^*, j^*) is the unique subgame-perfect equilibrium following this mechanism because j^* is a dominant strategy for the buyer. While the mechanism in the next subsection tries to mimic the above one, this feature will be lost under bilateral risk aversion.

b. Bilateral risk aversion

In this case, a unique starting point with one party as residual claimant may not suffice to implement the first best. Instead, we start with a message game G^*, which implements (p^*, q^*) if (i^*, j^*) has been chosen by the parties. From section III, this game exists under our assumptions. It turns out that it also prevents underinvestment. In order to avoid underinvestment, for each party we introduce a threat of starting from a very high output

level, q' and q".

Proposition 8 asserts that the first-best allocation corresponds the the outcome of a subgame-perfect equilibrium generated by such a contract. (There may however be other perfect equilibria, with different outcomes):

<u>Proposition</u> <u>8</u>: Call $\bar{\beta}$ and $\underline{\beta}$ (resp. $\bar{\sigma}$ and $\underline{\sigma}$) the lowest and highest values of β (resp. σ) ; call $\theta_{\ell'}$ the state defined by $(\bar{\beta},\underline{\sigma})$ and $\theta_{\ell''}$ the state defined by $(\underline{\beta},\bar{\sigma})$.[21] If $v(0,j,\beta)=0$ for any j and any β, and if the derivatives of $v(q,j,\beta)$ (resp. $c(q,i,\sigma)$) w.r.t. j and β (resp. i and σ) are unbounded when q increases, then there exists quantities q' and q" such that the following contract yields the first best as a perfect equilibrium outcome for any *interim* renegotiation process satisfying assumption 1: after the realization of the state of nature, do as follows:

- <u>Stage</u> <u>1</u>: The buyer can impose to have all the bargaining power and start from q' and $p'=c(q',i^*,\underline{\sigma})+S^*_{\ell'}$.
- <u>Stage</u> <u>2</u>: If the buyer abstains, the seller can impose to have all the bargaining power and start from q" and $p''=v(q'',j^*,\underline{\beta})-B^*_{\ell''}$.
- <u>Stage</u> <u>3</u>: If both parties abstain, G^* is played.

<u>Proof</u>: For (i^*,j^*), in state $\theta_{\ell'}$ the buyer is indifferent between G^* and the alternative he can impose in stage 1, whereas in state $\theta_{\ell''}$ the seller is indifferent between and his alternative. In all other

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21 If the two random variables β and σ are not independent, $\theta_{\ell'}$ corresponds to $(\tilde{\beta},\underline{\sigma})$, where $\tilde{\beta}$ is the highest β that coexists with $\underline{\sigma}$; similarly, $\theta_{\ell''}$ corresponds to $(\underline{\beta},\tilde{\sigma})$, where $\tilde{\sigma}$ is the highest σ to coexist with $\underline{\beta}$.

states, for q' and q'' large enough, each prefers G^* to his own alternative. For example the seller obtains exactly $S^*_{\ell'} \geq S^*_\ell$ in all states where $\sigma = \underline{\sigma}$ ($\theta_{\ell''}$ is the most favorable among those states) and, for q'' large enough, $\beta^*_\ell \leq v(q'', j^*, \beta_\ell) - p''$ whenever $\beta_\ell > \underline{\beta}$: playing G^* is thus again better for the seller.

Suppose that the above contract is not renegotiated before the investment decisions, and $(i,j)=(i^*,j^*)$. The contract would not then be renegotiated at the *interim* stage, since without renegotiation G^* would be played and would implement the (*interim* efficient) first-best allocation. It therefore suffices to show that the above contract, if not renegotiated before the investment decisions, would lead to (i^*,j^*).

Any departure from $(i,j)=(i^*,j^*)$ necessarily leads to an expected loss of surplus. A party can thus benefit from such a deviation only if it lowers the other party's expected utility below its first-best level, which in turn can happen, using Assumption 1 on *interim* renegotiation, only if the proposed scheme lowers this other party's *ex post* payoff below its first-best level for some state of nature. We show below that this cannot be the case for high enough quantities and adequate prices.

a) Overinvestment: we analyze the case of a buyer's overinvestment ($i=i^*, j>j^*$); the case of seller's overinvestment can be treated symmetrically.

If the buyer objects to G^* at stage 1, he has then all the bargaining power and his investment has no impact on the seller's default utility level, which by construction gives the seller at least S^*_ℓ. If the buyer abstains in the first stage, in state θ_ℓ the seller can guarantee himself at least S^*_ℓ by abstaining too, and

announcing θ_ℓ in G^*. His default utility level is then S_ℓ^* if the buyer also announces θ_ℓ. Otherwise, by construction of G^*: (i) the default options involve q=0, so that the corresponding utility levels are unaffected by the buyer's investment, and one party has all the bargaining power; (ii) If $j=j^*$ the seller would obtain at least S_ℓ^*. Since for $j>j^*$ the total surplus W is higher than W^*, the seller's payoff thus cannot decrease.

b) Seller's underinvestment ($i<i^*$ and $j=j^*$). For i less than i^*, in state $\theta_{\ell'}$ the buyer will impose (p',q'), and reduce the seller's payoff by $c(q',i,\underline{\sigma})-c(q',i^*,\underline{\sigma})$. On the other hand, lowering i has no impact on the buyer's payoff corresponding to the seller's alternative at stage 2; lastly, lowering i will have a negative impact on the buyer's expected payoff associated with G^* in states other than $\theta_{\ell'}$. This latter impact does not depend on q' or q''. Concerning the definition of q', for every $i \in [0,i^*[$, one can find $q'(i)$ big enough to allow the buyer to gain in expected term in comparison to i^*. Indeed, his loss in states other than $\theta_{\ell'}$ is given and bounded and so is the cut in total surplus in state $\theta_{\ell'}$ while, for $q'(i)$ arbitrarily big as default option with buyer power, the seller's payoff in state $\theta_{\ell'}$ tends to minus infinity. The function $q'(i)$ is bounded on $[0\ i^*[$ and has in particular a finite limit at i^*. Define q' as the supremum of $q'(i)$ on this interval. It will deter any seller's underinvestment (if the buyer chooses for some i's to impose (p',q') as starting point with full buyer power in states other than $\theta_{\ell'}$ too, this only reinforces the argument).

c) Buyer's underinvestment ($i=i^*,j<j^*$). The only difference with the case of seller;s underinvestment is that the buyer could

impose (p',q') in stage 1, instead of letting the seller impose (p'',q'') in stage 2 in state $\theta_{\ell''}$. However, for a given q'', this will not be profitable for q' large enough: imposing (p',q') in state $\theta_{\ell''}$ gives the seller $p'-c(q',i^*,\bar{\sigma})$; but for (p',q') satisfying $p'-c(q',i^*,\underline{\sigma})=S^*_{\ell'}$, the higher the q', the lower $p'-c(q',i^*,\bar{\sigma})$, since $\underline{\sigma}<\bar{\sigma}$. The procedure is thus to first find a q'' high enough so that $j<j^*$ would be too costly for the buyer if the seller imposes (p'',q'') in stage 2 in state $\theta_{\ell''}$. Then, q' is chosen to be high enough to simultaneously deter $i<i^*$ in part (b) above and induce the buyer to let the seller impose (p'',q'') in stage 2 when $\theta=\theta_{\ell''}$ (instead of imposing (p',q')).

Q.E.D.

The mechanisms described in this Section share a key feature: they treat underinvestment and overinvestment asymmetrically. Overinvestment is deterred because with G^* no player can extract more than the total amount of quasi-rents generated by any additional unit of his own investment -and in case of overinvestment the alternatives are not attractive. On the other hand, underinvestment is deterred by the threat of having to bargain from a very high output level, at which each unit of investment has an enormous impact on one's own payoff.

A final remark concerns our assumption on the form of risk aversion ((5.1) and (5.2)), i.e. the absence of wealth effects. It has allowed us to have only zero-output levels as starting points off-the-diagonal in matrix G^*. The only remaining problem was then underinvestment, and that was taken care of by threats prior to playing G^*. Instead, as seen in Section III, in the presence of wealth effects, optimal risk sharing may require very high output levels as starting points even without endogenous investment. The mechanisms

described above may thus be unable to achieve the first-best allocation. An analysis of this case is an interesting avenue for further research.[22]

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[22]An easy case is that of separable investment, i.e. $c(q,i,\sigma) = c(q,\sigma) - w(i)$, and $v(q,j,\beta) = v(q,\beta) + z(j)$, with $w', z' > 0$, and $w'', z'' < 0$. It is assumed that $w(i)$ and $z(j)$ are obtained only once the parties have reached an agreement, otherwise there would be no problem at all in reaching (i^*, j^*). Note that the mechanism constructed in Section 3 for exogenous investment still yields the exact same equilibria for any (i,j), since the outside option gives each party $w(i)$ or $z(j)$ anyway, and since a change in starting point (p,q) does not affect incentives to invest. There is thus no way in which for example $i \neq i^*$ can affect the buyer's expected payoff (or vice versa). It is thus a dominant strategy for each party to choose his first-best investment level. This result can be used to deal with the case of sufficiently convex investment cost functions ($\varphi(i)$ and $\psi(j)$) and sufficiently weak nonseparability (i.e. c_{qi} and v_{qj} close to zero). Indeed, consider a mechanism as in Section 3 which implements the first-best solution for $(i,j) = (i^*, j^*)$. For a finite number of strictly different states of nature, it is possible to make truthtelling a strict equilibrium of the message game in each state of nature. Under nonseparability, a unilateral investment deviation can prevent truthtelling from being an equilibrium of this game. However, for "small" unilateral deviations, this will not be true. But, then small deviations are not profitable: the starting points on the diagonal give the other party first-best payoff levels. And renegotiation from these starting points can only raise these payoffs. Not making your first-best investment choice can thus only reduce your own expected payoff when the other party chooses his first-best investment level. Finally, while the above argument relies on "small" investment deviations (larger ones might violate truthtelling in the original message game), larger investment deviations will not be worthwhile for $\varphi(i)$ and $\psi(j)$ sufficiently convex. The first-best outcome is thus a subgame-perfect equilibrium of this simple message game.

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APPENDIX A

In this Appendix, we generalize the extensive form of subsection II.3 in two respects:

(i) First, instead of alternating offers, we assume an exogenous function $f(n) \in \{B,S\}$, specifying the identity of the party making the offer in each renegotiation round from t_0 on (i.e., $f(n)$ determines who can make an offer at $t = t_0 + n\Delta$).

(ii) Second, we allow the following extensive form for each renegotiation round from t_2 on.

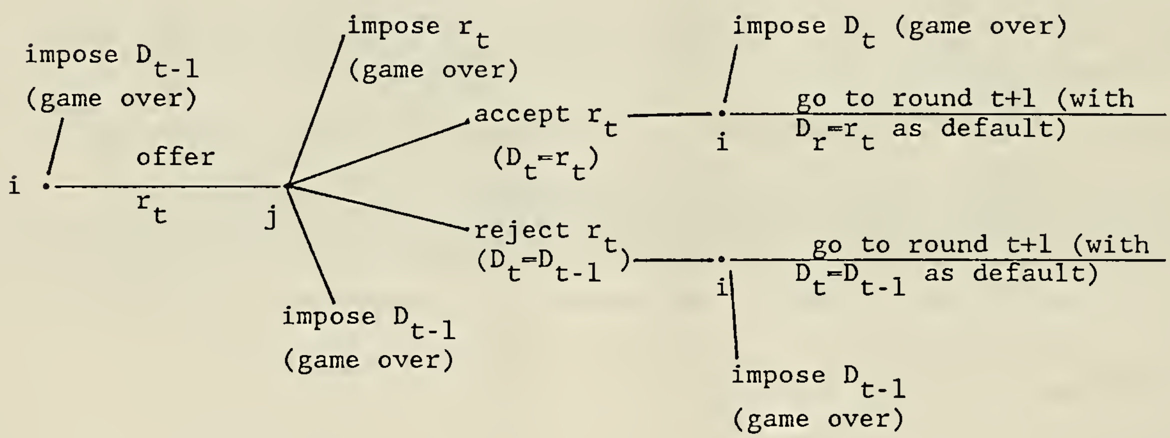

Figure 3′

In round $t = t_0 + n\Delta$, player i (selected by $f(\cdot)$) can either impose the default option relevant at that time (i.e. at the end of round t-1, so call it D_{t-1}), or make an offer r_t. The other player (j) can then either impose D_{t-1} or the offer r_t, or accept r_t as the new default option (D_t) (without

imposing it), or reject r_t. In the last two cases, i can talk again, and impose the relevant default option or wait until $t+\Delta$, time at which the next round starts. In all cases where an outcome is imposed the game ends with the utility levels associated to that outcome in this state of nature, discounted by δ_i^t and δ_j^t.

We could also drop the possibility for j to accept r_t simply as new default option D_t, and/or the possibility for i to impose D_{t-1} initially, and/or to talk again after j has decided not to end the game by imposing D_{t-1} or r_t.

In this more general formulation, assumptions 0 and 1 may not be satisfied. However, the introduction of renegotiation design allows us to get a result equivalent to Proposition 1. Specifically, we introduce the possibility of a transfer T, and also the possiblity for one party to offer the other one, in the last verifiable message stage, a second default option. Our assumption on preferences w.r.t. T is somewhat stronger.

<u>Assumption (I′)</u>: $\forall i \neq j,\ i,j \in \{B,S\},\ \forall\ (p,q),\ \exists T \in \mathbb{R}$ s.t. $\forall \theta \in \Theta$,

$$\operatorname{Min}\ \{U_i(T,0,\theta),\ \delta_i U_i(p+\rho_i T,q,\theta)\} \geq \operatorname*{Max}_{\tilde{p},\tilde{q}\ \text{s.t.}\ U_j(\tilde{p},\tilde{q},\theta) \geq U_j(p,q,\theta)} U_i(\tilde{p},\tilde{q},\theta),$$

where ρ_i is such that $e^{-\rho_i \Delta} = \delta_i$.

The idea is that T is not only big enough so that the status quo dominates the best outcome i could get while giving j $U_j(p,q,\theta)$, but also that i would prefer waiting and receiving (p,q) tomorrow (plus the interest on T) instead of this best outcome today (without the interest on T thus).

Under assumption (I′), we have the following result:

<u>Proposition 1'</u>: Assume the game until t_2-Δ has yielded a default option (p,q), and assumption (I') is satisfied. Then, there exist an appropriate transfer level, and an additional default option, contingent on the state of nature, so that the unique subgame-perfect equilibrium outcome from t_2 on is *ex post* efficient in each state of nature and gives one party the utility associated with (p,q).

<u>Proof</u>: Assume for example one wants to allocate all bargaining power to the seller (i.e. give the buyer $U_B(p,q,\theta)\ \forall\theta$). Then, find T big enough so that $\forall\theta$, $\min(U_S(t,0,\theta),\ \delta_S U_S(p+\rho_S T,q,\theta)) \geq \underset{\tilde{p},\tilde{q}}{\mathrm{Max}}\ U_S(\tilde{p},\tilde{q},\theta)$ s.t. $U_B(\tilde{p},\tilde{q},\theta) \geq U_B(p,q,\theta)$. Such T exists by assumption I'. T is a hostage paid by the buyer to the seller at t_2, and refunded whenever trade occurs. T thus changes the status quo point, and moves it to a higher utility level for the seller in each state of nature than the best he can get without lowering the buyer's payoff below its default option level.

Now, allow the seller to pick a second default option for the buyer in the last verifiable message stage. Then, the seller can in fact have full bargaining power: He can offer the buyer an *ex post* efficient (p',q') in state θ such that $U_B(p',q',\theta) = U_B(p,q,\theta)+\epsilon$, i.e. making in effect (p,q) dominated for the buyer by (p',q'). For ϵ arbitrarily small, this outcome would give the seller all bargaining power. Is it the unique subgame-perfect equilibrium from t_2 on? Certainly, the buyer can guarantee himself this utility level. Moreover, (p',q') is the worst that can happen to the seller, since (p,q) is dominated for the buyer, and the seller can prevent the buyer from imposing anything worse than (p',q') then. Finally,

he prefers the status quo to (p′,q′). The optimal strategy for the buyer is thus to impose (p′,q′) immediately because, by assumption (I), the seller only gains by waiting. Allocating all bargaining power to the buyer works exactly symmetrically (and requires T low enough). Q.E.D.

In terms of Figure 5, we give the ability to seller to give the buyer an additional default option S′ (contingent on θ, since this happens once θ is known) which is *ex post* efficient and arbitrarily close but to the right of S′. This makes (p,q) dominated for the buyer by S′, and allows the seller to get all bargaining power (instead, in the absence of S′, as shown by Shaked (1987)), the buyer could in some cases ask for an *ex post* efficient outcome slightly to the left of B, and threaten the seller to impose (p,q) if it is rejected.

The conclusion of this Appendix is thus that assumptions 0, 1 and 2 can be derived in a fairly general sequential bargaining framework, provided one allows the parties to impose default options and to make transfers refundable upon agreement.

APPENDIX B

Proposition 2′: (i) If an allocation r^* is implementable, then for any possible pair of states of nature (θ,θ') there exists a probability distribution $P_{\theta\theta'}$ over $\{B,S\} xR$ such that:

$$E_{P_{\theta\theta'}}[\tilde{U}_S(\alpha,r,\theta)] \le U_S(r^*_\theta,\theta) \text{ and } E_{P_{\theta\theta'}}[\tilde{U}_B(\alpha,r,\theta')] \le U_B(r^*_\theta,\theta').$$

If moreover there exists a perfect equilibrium implementing r^* in which equilibrium strategies are pure, then the probability functions can be chosen degenerate, i.e. for any pair of states of nature (θ,θ') there exists a game $(\alpha_{\theta\theta'},r_{\theta\theta'})$ such that:

$$\tilde{U}_S(\alpha_{\theta\theta'},r_{\theta\theta'},\theta) \le U_S(r^*_\theta,\theta) \text{ and } \tilde{U}_B(\alpha_{\theta\theta'},r_{\theta\theta'},\theta') \le U_B(r^*_\theta,\theta').$$

Conversely, if an allocation r^* is *ex ante* efficient and if for any pair of states of nature there exists a game $(\alpha_{\theta\theta'},r_{\theta\theta'})$ which has the above properties, then:

(ii) if only pure strategies are allowed, the allocation r^* can be (truthfully) implemented using a one-stage direct revelation game[1] which assigns, to any pair of messages (θ_B,θ_S) sent by the agents, the game $(\alpha_{\theta_B\theta_S},r_{\theta_B\theta_S})$ - with the convention that $r_{\theta\theta}=r^*_\theta$.

(iii) if mixed strategies are considered as well, the allocation r^* can be (truthfully) implemented by a direct revelation game which is the same as in (ii), except that the announcements are sequential (one agent first announces a state of nature, and then the other agent, having observed the first claim, announces a state of nature); the allocation r^* can alternatively be implemented using a one-stage (possibly indirect) game.

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[1] I.e, both agents must simultaneously announce a state of nature, announcing the truth is an equilibrium and all equilibrium payoffs coincide with those of the allocation r.

Proof: We first show that probability distributions can indeed be associated with any implementable allocation as indicated in part (i). We will then check that the mechanisms described in parts (ii) and (iii) indeed implement the corresponding allocation if this allocation is ex ante efficient.

(i) Note first that the previous lemma allows us to consider truncated games, where each subgame starting from t_2 is replaced by efficient payoffs corresponding to some (α, r) in $\{B,S\}xR$. Therefore, if an allocation is implementable, starting from t_1 there exists such a game and a subgame perfect equilibrium of this game which leads to the same payoffs as this allocation.

Consider now the normal form of this game, and let b_θ and s_θ respectively denote the buyer's and seller's equilibrium (possibly mixed) strategies when the state of nature is θ; then the probability $P(\theta,\theta')$ induced by the pair of strategies $(b_\theta, s_{\theta'})$ satisfies the property announced in the proposition. Moreover, by construction, if the two players equilibrium strategies are pure, then the probability distributions $P_{\theta\theta'}$ are degenerate, which concludes the proof of the part (i).

(ii) It is straightforward to check that truthfully announcing the state of nature is an equilibrium of the game proposed in part (ii). Let $\tilde{r}_\theta(\theta_B, \theta_S)$ denote the payoff associated in state θ with the pair of announcements (θ_B, θ_S).[2] By construction in state θ the buyer (at least weakly) prefers $\tilde{r}_\theta(\theta,\theta) = r^*_\theta$ to any other $\tilde{r}_\theta(\theta_B, \theta)$ and similarly the seller prefers $\tilde{r}_\theta(\theta,\theta) = r^*_\theta$ to any other $\tilde{r}_\theta(\theta, \theta_S)$.

If only pure strategies are allowed, all equilibria in this game have the same payoffs. Suppose for instance that in state θ, there exists two

[2]I.e. $\tilde{r}_\theta(\theta_B, \theta_S)$ is in state θ the unique equilibrium outcome of the game $(\alpha_{\theta_B\theta_S}, r_{\theta_B\theta_S})$

different equilibria, (θ,θ) and (θ_B,θ_S), which implies that the seller prefers (weakly) $\tilde{r}_\theta(\theta,\theta)$ to $\tilde{r}_\theta(\theta,\theta_S)$ and $\tilde{r}_\theta(\theta_B,\theta_S)$ to $\tilde{r}_\theta(\theta_B,\theta)$, whereas the buyer prefers $\tilde{r}_\theta(\theta,\theta)$ to $\tilde{r}_\theta(\theta_B,\theta)$ and $\tilde{r}_\theta(\theta_B,\theta_S)$ to $\tilde{r}_\theta(\theta,\theta_S)$; since all payoffs are efficient given θ, these buyer's preferences imply reversed preferences for the seller, which therefore prefers $\tilde{r}_\theta(\theta_B,\theta)$ to $\tilde{r}_\theta(\theta,\theta)$ and $\tilde{r}_\theta(\theta,\theta_S)$ to $\tilde{r}_\theta(\theta_B,\theta_S)$: all together, this implies that the seller is indifferent between $\tilde{r}_\theta(\theta,\theta)=r^*(\theta)$ and $\tilde{r}_\theta(\theta_B,\theta_S)$ -and thus so is the buyer.

Thus if the proposed game is played at some date t after t_1, it implements the allocation r^*. Between t_1 and this date t, and conditional on the state of nature, both agents can guarantee themselves the payoffs associated with the corresponding efficient outcome r^*_θ by "waiting" (i.e. proposing no new offer and refusing any new offer) until this game is played; therefore if at date t_1 the current contract specifies that the above game must be played at some date between t_1 and t_2, all equilibrium payoffs from t_1 will coincide with those of the allocation r^*.

Lastly, if the allocation r is also *ex ante* efficient, then the same argument can be applied to renegotiation rounds between t_0 and t_1, and thus a contract, signed at t_0, which specifies that the game proposed in (ii) has to be played at some date between r_1 and r_2 implements the allocation r^*.

(iii) Consider now the game proposed in part (iii), where the buyer, say, is the first to speak, and allow for mixed strategies. In state θ, if the buyer announces θ_B, all announcements in the support of the seller's best response[3] are (weakly) preferred by the seller to θ. Therefore, announcing θ_B leads to an outcome which cannot be better for the buyer than $\tilde{r}_\theta(\theta_B,\theta)$, which by construction is weakly worse than $\tilde{r}_\theta(\theta,\theta)$ for the buyer. Conversely, if the buyer announces θ, then by construction θ is a seller's

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[3] We implicitly assume that there exists a seller's best response; this would be for instance be satisfied under some compactness assumptions on the set of characteristics Θ and the set of *ex post* efficient payoffs.

best response : the seller prefers $\tilde{r}_\theta(\theta,\theta)$ to any other $\tilde{r}_\theta(\theta,\theta_S)$. Moreover, as already mentioned, all possible seller's best responses lead to the same payoffs. Therefore, by announcing θ, the buyer obtains for sure the payoff associated with $\tilde{r}_\theta(\theta,\theta)=r^*_\theta$; θ is thus a best strategy for the buyer, and by the same argument as before all buyer's best strategies lead to the same payoffs.

This shows that, whether mixed strategies are allowed or not, all perfect equilibria of the game proposed in (iii) have the same payoffs as the allocation r^*. As in (ii), if this allocation is ex ante efficient, then at t_0, a contract which specifies that this game has to be played at, say, dates t_1^a and t_2^a, indeed implements this allocation (one can easily check that introducing renegotiation between the two announcement stages does not not alter the above reasoning).

Q.E.D.